

Surface Stress and Microstructural Effects on Elastic Substrate Contact Problems

Jintara Lawongkerd,^{1,*} Suraparb Keawsawasvong,² Suchart Limkatanyu,³ Jaron Rungamornrat^{4,5,*} and Punyatorn Jangkrajang¹

¹*Research Unit in Advanced Mechanics of Solids and Vibration, Department of Civil Engineering, Thammasat School of Engineering, Faculty of Engineering, Thammasat University, Pathumthani, 12120, Thailand*

²*Department of Civil Engineering, Faculty of Engineering, Thammasat School of Engineering, Thammasat University, Pathumthani, 12120, Thailand*

³*Department of Civil Engineering, Faculty of Engineering, Prince of Songkla University, Songkhla, 90110, Thailand*

⁴*Center of Excellence in Applied Mechanics and Structures, Department of Civil Engineering, Faculty of Engineering, Chulalongkorn University, Bangkok, 10330, Thailand*

⁵*Center of Digital Asset Management for Sustainable Development, Department of Civil Engineering, Faculty of Engineering, Chulalongkorn University, Bangkok, 10330, Thailand*

*Email: ljintara@engr.tu.ac.th (J. Lawongkerd); Jaron.r@chula.ac.th (J. Rungamornrat)

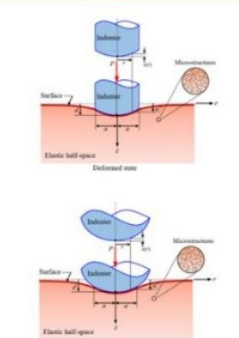
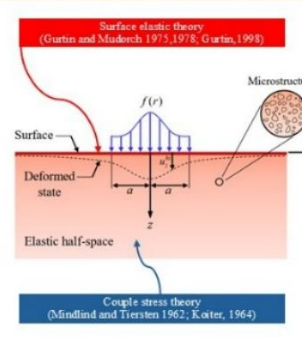
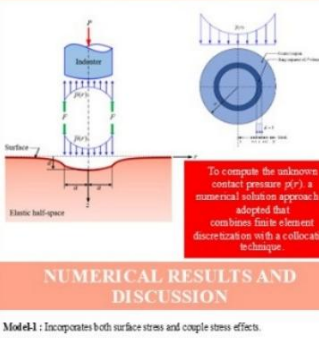
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Abstract

This work presents an elastic contact model for a half-space indented by a rigid, axisymmetric indenter under frictionless conditions, explicitly accounting for the influence of surface energy and intrinsic material microstructures. The mechanical response of the bulk substrate is characterized using couple stress theory, while surface elasticity theory governs the behavior of the surface layer. The contact pressure distribution is obtained by formulating a governing equation based on force equilibrium and displacement compatibility within the contact region. A combination of finite element discretization, a collocation approach, and the linearity of the governing equations is employed to solve for the unknown pressure profile. The Hankel transform technique is used to analytically derive the fundamental solutions for surface displacement. The contact radius is determined through an iterative bisection algorithm. Comprehensive numerical simulations validate the proposed methodology and highlight the critical impact of both couple stresses and surface effects on the contact response. In particular, the results reveal pronounced size effects when the internal and external length scales are of similar magnitude.

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<p>PROBLEM DESCRIPTION</p>  <p>This study develops a theoretical and computational framework to analyze the elastic response of a half-space indented by a rigid, axisymmetric indenter. The model incorporates couple stress and surface elasticity theories to account for microstructural effects and surface energy. The contact pressure $p(r)$ is treated as the primary unknown, and the solution is obtained through a collocation-based numerical strategy.</p>	<p>BASIC FIELD EQUATIONS AND INTERFACE CONDITIONS</p>  <p>Surface elastic theory (Gurtin and Murdoch 1975, 1978; Gurtin, 1998)</p> <p>Couple stress theory (Mindlin and Tiersten 1962; Koster, 1964)</p> <p>The combination of the two theories together provides a complete set of equations to model an elastic half-space subjected to surface excitations.</p>	<p>FORMULATION OF INDENTATION PROBLEMS</p>  <p>To compute the unknown contact pressure $p(r)$, a numerical solution approach is adopted that combines finite element discretization with a collocation technique.</p> <p>NUMERICAL RESULTS AND DISCUSSION</p> <p>Model-1: Incorporates both surface stress and couple stress effects.</p> <p>Model-2: Includes only surface stress effects (i.e., $\mu', \mu'', \nu' \rightarrow 0$).</p> <p>Model-3: Includes only couple stress effects (i.e., $\mu', \mu'', \nu' \rightarrow 0$).</p> <p>Model-4: Represents the classical elasticity model, where both effects are neglected (i.e., $\ell, \ell', \mu', \mu'', \nu' \rightarrow 0$).</p>

Innovative Description: Forming governing equations via indenter equilibrium and surface-displacement compatibility