



# Identification of Resistance Coefficients for Section of Water Pipeline Network in Case of Intermediate Withdrawals or Injections of Water

Timur Bekibayev\*

## Abstract

This study introduces a new approach for identifying hydraulic resistance coefficients (roughness, minor losses, and friction factor) in pipeline network sections with intermediate water withdrawals or injections, using only endpoint measurements. The approach combines the Colebrook-White equation for turbulent flow modeling with stochastic methods to address parameter variability and uses regression analysis to derive equivalent friction factor relationships, utilizing only endpoint pressure, flow rate, and temperature measurements. Analytical formulas were developed to determine resistance coefficients without solving complex nonlinear systems. The method's efficacy was validated through application to the Astrakhan-Mangyshlak pipeline (Kazakhstan), demonstrating accurate parameter estimation under real-world conditions. The developed approach enables accurate identification of resistance coefficients through simplified endpoint-only measurements, providing a computationally efficient alternative to traditional methods that can be directly applied to operational diagnostics in large-scale pipeline networks with limited sensor coverage.

**Keywords:** Water pipeline; Friction factor; Branch points; Water withdrawal; Colebrook-White equation.

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## 1. Introduction

Nowadays, the pumping of water through pipeline networks is of great economic and social importance. For example, it is used for urban systems of district heating and water supply, and for trunk water pipelines, which transport fresh water for hundreds of kilometers. In the age of digitalization, various software tools for simulating and optimizing the above water pumping processes play an important role. Simulation of these processes can determine efficient and accident-free control modes. Simulation of water pumping through pipeline networks requires accurate calculation of hydraulic losses in pipes, because hydraulic resistance of pipes are necessary parameters for calculation of flow distribution.<sup>[1]</sup> As a rule, calculation of hydraulic losses uses hydraulic resistance coefficients such as pipe surface roughness and coefficient of minor losses influence or actual dependence of friction factor on the Reynolds number.

Pipeline resistance changes over many years of its operation.<sup>[2-5]</sup> It is known that pipe roughness is increasing over years. In the case of water pumping, inevitable corrosion processes occur on the inner wall of the pipe,<sup>[6]</sup> which increase the roughness coefficient. Also, over many years of operation there may be various deposition along the inner wall of the pipe such as sediment (sand, silt),<sup>[7,8]</sup> or calcium carbonate,<sup>[9]</sup> it also changes pipe roughness and may significantly decrease diameter of cross-sectional area of water flow. Hence, deposition along the pipe's inner wall causes additional pressure losses. The spontaneous formation of limescale (CaCO<sub>3</sub>) on pipelines affects the efficiency of water supply networks.<sup>[9,10]</sup> Therefore, in practice, it is quite problematic to accurately identify pipe roughness and the coefficient of minor losses influence. Without considering these facts and using only the reference hydraulic parameters of the pipes, the calculation of hydraulic losses may have a significant error.

Problems of pipe resistance coefficients identification are considered in papers.<sup>[11-17]</sup> These papers have various problem statements and their approaches have various complexity levels of solutions. Authors of papers identify resistance coefficients for simple pipe section without intermediate

*Scientific and Production Laboratory "Energy Modeling", Institute of Automation and Information Technologies, Satbayev University, Almaty, 050013, Kazakhstan*

\*E-mail: [timur\\_bekibaev@mail.ru](mailto:timur_bekibaev@mail.ru) (T. Bekibayev)

withdrawals or injections of fluid (*i.e.*, with constant flow rate along pipe length).<sup>[11-14]</sup> The roughness and minor losses' coefficients are identified using nonlinear optimization.<sup>[11,12]</sup> Bekibayev *et al.* identify friction factor's dependence on the Reynolds number for oil pipeline using actual points of measured friction factors.<sup>[13]</sup> The roughness coefficient of gas pipeline is identified using simple iterations of its value which gives the lowest difference of calculated and actual values of pressures at section's end.<sup>[14]</sup> Authors of papers consider the whole pipelines networks.<sup>[15-17]</sup> In this case measurement data of the node pressure (or heads) or pipe flow rates were assumed as the known variables. The relationship between the known and unknown variables (including pipe resistance coefficients) were determined using the Kirchhoff equations. As result, these approaches require solution of a complex system of nonlinear equations (due to nonlinear dependence of pressure drop on liquid flow rate). Thereby in the systems of nonlinear equations are solved using modified Newton-Raphson approach or successive linearization method.<sup>[15-17]</sup>

Resistance coefficients of a pipeline, which has a constant flow rate along its length, can be calculated using measurements of pressure and flow rate at its ends. However, in practice measuring nodes are located in such a way that pipeline sections between 2 measuring nodes may contain multiple branch points which withdraw or inject water (see Fig. 1). Generally, branch points withdraw water from water supply systems, feed lines of the district heating system, and trunk water. Pipeline branch points inject water in sewage systems and return lines of district heating system. Pipeline resistance coefficients are especially important in simulation of predictive modes such as connection of new consumers or disconnection of previous consumers.

As a rule, water pipeline networks contain a large number of branch points and a comparatively small number of

measuring nodes. In practice, resistance coefficients of pipelines cannot be measured directly; instead, they are estimated indirectly, which may pose challenges in determining them in sections of the pipeline with branch points.

Despite extensive research on water pipeline networks, new challenges continue to emerge due to rapid urban growth and aging infrastructure. Cities are expanding, new buildings and entire residential districts are being constructed, and new pipeline branches are being integrated into existing networks. In this dynamic environment, accurately and efficiently redistributing water consumption within the network becomes essential. This redistribution requires highly precise hydraulic calculations, particularly for district heating systems, where operating conditions are sensitive to flow resistance.

However, most existing methods for identifying hydraulic resistance coefficients face two major limitations: (1) they rely on solving complex systems of nonlinear equations, which often suffer from issues, such as, sensitivity to initial conditions and numerical instability, and (2) they require extensive instrumentation throughout the network, which is impractical in many real-world situations.

To address these shortcomings, the present study introduces an alternative approach that uses scalar coefficients ( $\alpha$  and  $\beta$ ) to replace the more complex vector-based formulations of branch locations and intermediate lateral flow rates. This simplification allows for the exclusion of nonlinear systems from the identification process. By applying stochastic modeling and regression analysis, the method produces analytical expressions for the friction factor as a function of Reynolds number, based solely on endpoint sensor data. This not only reduces computational complexity but also expands the potential for practical applications in pipeline network analysis and software development.

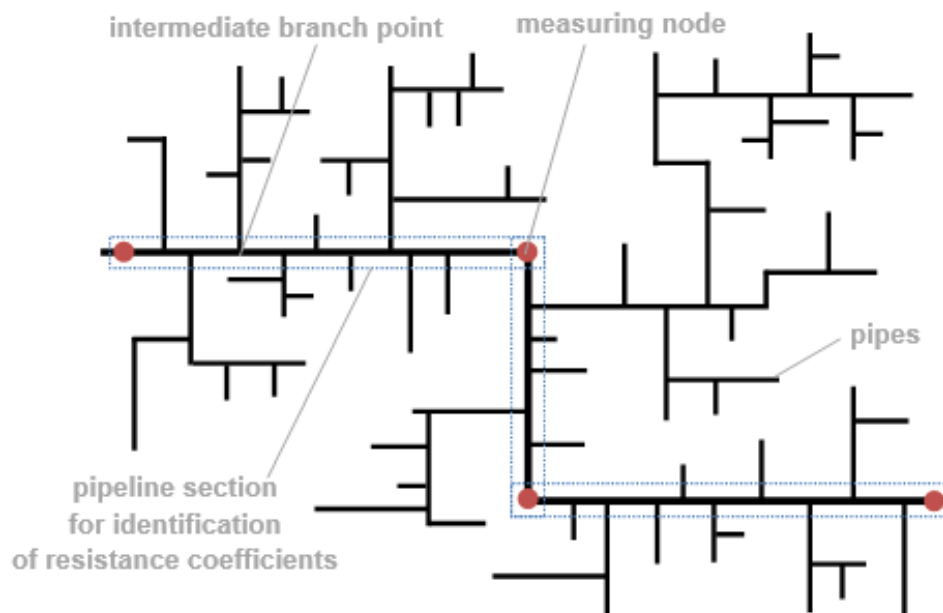


Fig. 1: Measuring nodes and pipeline network sections.

Therefore, the aim of this study is to develop a method for identifying pipeline network resistance coefficients with intermediate withdrawal/injection points using only endpoint measurement data.

**2. Problem statement**

Let there be a section of pipeline network with a circular cross section, inner diameter  $D$  and length  $L$ . Through the pipeline section water is pumped under pressure. The water has density  $\rho$  and kinematic viscosity  $\nu$  which depend on temperature. At the inlet and outlet of the pipeline section, the geodesic heights of the pipe axis are  $h^{in}$  and  $h^{out}$ , respectively. The pipeline section has  $N$  measurements (or known data) at different time moments, and each  $i$ -th measurement contains the following data (see Fig. 2).

- Inlet and outlet pressures,  $P_i^{in}$  and  $P_i^{out}$ ;
- Inlet and outlet volumetric flow rates,  $Q_i^{in}$  and  $Q_i^{out}$  (values are more than zero);
- Temperature of water,  $T_i$ .

In this issue, the pumped water has an ambient temperature or temperature defined by heat regime (in case of district heating system). Therefore, thermal conduction of pumped water with the environment is neglected. The value of  $T_i$  is assumed as constant along the length  $L$ .

Since water has comparatively low values of viscosity, it makes sense to consider only turbulent regime for water flow in the pipeline section (because of  $Re \gg 4000$ ).

The pipeline section contains  $M$  branch points. Each  $j$ -th branch point ( $1 \leq j \leq M$ ) is located at  $x_j$  on the pipeline section. Let  $x_0 = 0$ ,  $x_{M+1} = L$ . Let for some moment after each  $j$ -th branch point the flow rate changes by  $q_j$  and  $\sum_{j=1}^M q_j = Q^{in} - Q^{out}$ . In case of  $q_j > 0$  ( $1 \leq j \leq M$ ) branch points withdraw water. In case of  $q_j < 0$  ( $1 \leq j \leq M$ ) branch points inject water. Let for this moment define a function  $F(x)$  as follows:

$$F(x) = \begin{cases} 0, & 0 \leq x < x_1 \\ \frac{1}{Q^{in}-Q^{out}} \sum_{j=1}^j q_j, & x_j \leq x < x_{j+1} \\ 1, & x = L \end{cases} \quad (1)$$

In Eq (1),  $F(x)$  is cumulative distribution of relative volume of water withdrawals (or injections). In other words, function  $F(x)$  determines withdrawn or injected volume of water before reaching  $j$ -th branch point as  $F(x_j) \cdot (Q^{in} - Q^{out})$ . As a rule, water pipeline networks supply consumers of different capacities and pipeline’s branch points cover a different number of consumers (see branches on Fig. 1). Therefore, this issue considers non-uniform distribution of withdrawals (or injections) along the pipeline section (see Fig. 2). As a rule, in urban systems of district heating and water supply, absolute volumes of consumers’ water withdrawals (or injections) are various and depend on time of day (in case of water supply) or on time of year (in case of district heating), but relative volumes of consumers’ water withdrawals (or injections) are more or less constant in time. Therefore, for this issue the function  $F(x)$  is assumed as immutable in time.

It is commonly known that hydraulic pressure losses  $\Delta P$  in pipeline are caused by friction loss  $\Delta P^L$  along its length (linear losses) and minor losses. Let define expression for  $\Delta P$  considering that value  $\Delta P^L$  is determined by the Darcy-Weisbach equation.<sup>[18,19]</sup>

$$\Delta P = K \cdot \Delta P^L = K \cdot \lambda_D \frac{l u^2}{D} \rho = \lambda \frac{l u^2}{D} \rho \quad (2)$$

where  $K$  is coefficient of minor losses influence (elbows, tees, gate valves, U-bends, etc.), function  $\lambda_D = \lambda_D(Re, \frac{\epsilon}{D})$  is the Darcy friction factor that depends on the Reynolds number  $Re$  and effective roughness height  $\epsilon$  of pipe inner wall,  $l$  is a length of pipe,  $u$  is average flow velocity on the cross-sectional area of the pipe. Let’s call quantity  $\lambda = K \cdot \lambda_D$  of expression Eq. (2) as generalized friction factor ( $\lambda$  introduces

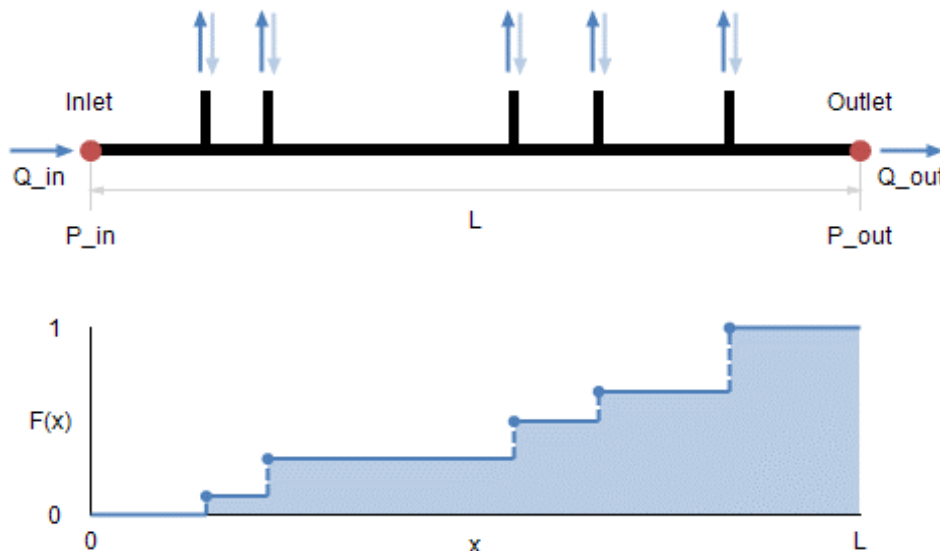


Fig. 2: The section of pipeline network and function  $F(x)$ .

also minor losses). This issue requires identification of function  $\lambda = \lambda(Re)$  and values  $\varepsilon, K$  for the above-described pipeline section using actual values of  $P_i^{in}, P_i^{out}, T_i, Q_i^{in}, Q_i^{out}$  ( $1 \leq i \leq N$ ).

Cross-sectional area of water flow may differ significantly from cross-sectional area with pipe's inner diameter  $D$  (because of probable silting on pipe's inner wall). Real diameter of water flow area is unknown. Hence, flow velocity for expression Eq. (2) and the Reynolds numbers for the function  $\lambda = \lambda(Re)$  are defined by using of the given diameter  $D$ .

### 3. Proposed approach

Let's construct definitions of points  $(Re, \lambda)$  for the function  $\lambda = \lambda(Re)$  by measured data of flow rates and pressures assuming that in the pipeline segment real diameter of water flow is equal to given pipe's inner diameter  $D$ . And then let's provide determination of function  $\lambda = \lambda(Re)$  and identification of  $\varepsilon, K$  using calculated points  $(Re, \lambda)$ . At the end of this chapter there is demonstrated that imprecisions of given  $D$  influences only the coefficients  $K$  and  $\varepsilon$ .

#### 3.1 Determination of points for the function $\lambda = \lambda(Re)$

Let's at first examine a known case of constant flow rate along the pipeline section (simple case) in order to describe formulae for actual points  $(Re, \lambda)$  of dependence  $\lambda = \lambda(Re)$ . According to formulae of the simple case, let's construct formulae for determination of «equivalent» points  $(Re^*, \lambda^*)$  of dependence  $\lambda = \lambda(Re)$  for a case of water withdrawals. Then let's demonstrate that a case of water injections can be reduced to the case of water withdrawals.

##### 3.1.1 Simple case

Let's consider  $q_j = 0$  ( $1 \leq j \leq M$ ). In this case water flow rate is constant along the pipeline section, consequently  $Q^{in} = Q^{out} = Q$ .

In practice, value of hydraulic pressure losses along the section  $L$  can be defined as difference of total pressure drop and static pressure drop:

$$\Delta P = P^{in} - P^{out} - \rho g(h^{in} - h^{out}) \quad (3)$$

In accordance with expression  $u = \frac{4Q}{\pi D^2}$  (definition of average flow velocity of circular pipe), expression  $Re = \frac{uD}{\nu}$  (definition of the Reynolds number) and Eqs. (2) and (3) – actual values of  $Re_i, \lambda_i$  for  $i$ -th measurement of the pipeline section are defined as follows:

$$Re_i = \bar{Re}(Q_i) = \frac{4Q_i}{\nu(T_i)\pi D} \quad (4)$$

$$\lambda_i = \bar{\lambda}(Q_i) = \Delta P_i \frac{\pi^2 D^5}{8L\rho Q_i^2} = \left( P_i^{in} - P_i^{out} - \rho g(h^{in} - h^{out}) \right) \frac{\pi^2 D^5}{8L\rho Q_i^2} \quad (5)$$

Hence, if  $Q^{in} = Q^{out} = Q$ , then actual points  $(Re_i, \lambda_i)$  for the function  $\lambda = \lambda(Re)$  can be calculated using actual values of  $P_i^{in}, P_i^{out}, T_i, Q_i$  and Eqs. (4) and (5).

#### 3.1.2 Case of water withdrawals

Let's consider  $q_j > 0$  ( $1 \leq j \leq M$ ). In this case water flow rate decreases along the pipeline section, consequently  $Q^{in} > Q^{out}$ , and branch points are considered as water withdrawal points.

In case of use of withdrawal points' values  $[q_1, q_2, \dots, q_M]$  for calculation of intermediate flow rates ( $Q = Q^{in} - \sum q_j$ ) between withdrawal points, direct calculation of actual points  $(Re_{ij}, \lambda_{ij})$  for  $\lambda = \lambda(Re)$  is impossible because according to the problem statement there are unknown pressure values at withdrawal points  $[x_1, x_2, \dots, x_M]$ . Therefore, it is necessary to propose another method of calculation of points for  $\lambda = \lambda(Re)$ . For this purpose, let's define additional coefficients for the pipeline section:

$$\alpha = \frac{Q^{in} - Q^{out}}{Q^{in}}, \beta = \frac{1}{L} \int_0^L F(x) dx = \frac{1}{L} \sum_{j=1}^M F(x_j)(x_{j+1} - x_j) \quad (6)$$

where the coefficient  $\alpha$  determines the related volume of decrease in the inlet flow rate,  $0 < \alpha < 1$ .

The coefficient  $\beta$  depends on the proportion of withdrawn water volume regarding the pipeline section's beginning as opposed to its end. If withdrawn water is uniform regarding the beginning and the end of the pipeline section, then  $\beta = 0.5$  (see Fig. 3a). If water is withdrawn more in the section's beginning, then  $\beta > 0.5$  (see Fig. 3b), otherwise  $\beta < 0.5$  (see Fig. 3c). The more proportion of withdrawn water is in the section's beginning, the more value of  $\beta$  is, that is  $\beta \rightarrow 1$ . The more proportion of withdrawn water is in the section's end, the less value of  $\beta$  is, that is  $\beta \rightarrow 0$ . According to definitions of  $\beta$  and  $F(x)$ ,  $0 < \beta < 1$ . By this means, the variety of water withdrawal parameters (vectors  $[x_1, x_2, \dots, x_M]$  and  $[q_1, q_2, \dots, q_M]$ ) is described by only two the above defined coefficients  $\alpha$  and  $\beta$ . Fig. 3 shows examples of the functions  $F(x)$  for various values of  $\beta$ .

Let's define an «equivalent» case for the pipeline segment that characterize its case with withdrawals. Let «equivalent» case has the same value  $\Delta P$  (as in the case with withdrawals), but has no withdrawals, its flow rate  $Q^*$  is constant along the pipeline segment. It is evident that  $Q^{out} < Q^* < Q^{in}$ . A point  $(Re^*, \lambda^*)$  of «equivalent» case (that defined by Eqs. (4) and (5) using argument  $Q^*$ ) belongs to the function  $\lambda = \lambda(Re)$  of the pipeline section. According to Eqs. (3)-(5) there are correct inequalities  $\bar{Re}(Q^{out}) < Re^* < \bar{Re}(Q^{in})$  and  $\bar{\lambda}(Q^{in}) < \lambda^* < \bar{\lambda}(Q^{out})$ . In other words, the function  $\lambda = \lambda(Re)$  with arguments in range  $[\bar{Re}(Q^{out}), \bar{Re}(Q^{in})]$  has values in range  $[\bar{\lambda}(Q^{in}), \bar{\lambda}(Q^{out})]$ . This area is shown on Fig. 4 by orange color. Value  $Re^*$  of «equivalent» point can be represented as a linear combination of  $\bar{Re}(Q^{in})$  and  $\bar{Re}(Q^{out})$ , and value  $\lambda^*$  can be represented as a linear combination of  $\bar{\lambda}(Q^{in})$  and  $\bar{\lambda}(Q^{out})$ :

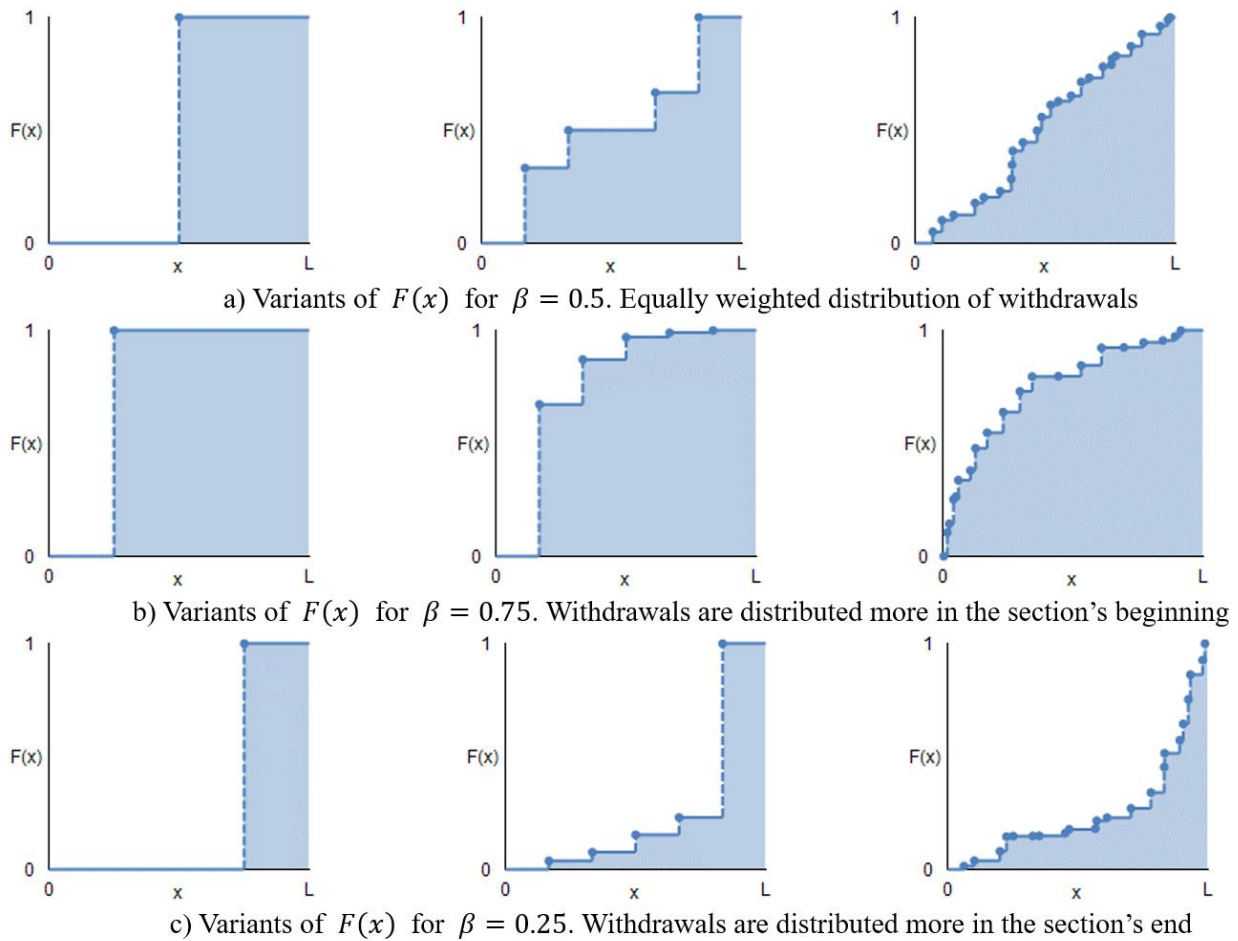


Fig. 3: Examples of the functions  $F(x)$  for various values of  $\beta$ .

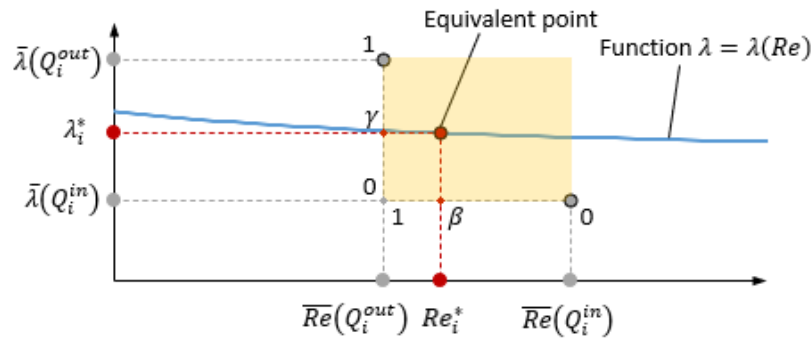


Fig. 4: Equivalent point and points calculated using inlet and outlet flow rates.

$$Re^* = (1 - c_{Re}) \cdot \overline{Re}(Q^{in}) + c_{Re} \cdot \overline{Re}(Q^{out}), \quad 0 < c_{Re} < 1 \quad (7)$$

$$\lambda^* = (1 - c_\lambda) \cdot \bar{\lambda}(Q^{in}) + c_\lambda \cdot \bar{\lambda}(Q^{out}), \quad 0 < c_\lambda < 1 \quad (8)$$

Let's define coefficients  $c_{Re}$  and  $c_\lambda$  of these linear combinations. It is evident that  $c_\lambda$  somehow depends on  $c_{Re}$ . Coefficient  $c_{Re}$  can be defined in an arbitrary way with the condition  $0 < c_{Re} < 1$ , because  $Re^*$  of «equivalent» point can have any value in the range  $[\overline{Re}(Q^{out}), \overline{Re}(Q^{in})]$  (see Fig. 4). There was suggested a hypothesis that dependence of  $c_\lambda$  on  $c_{Re}$  can be expressed through only two parameters (besides  $c_{Re}$ ): suggested coefficients  $\alpha, \beta$  which describe branch points

parameters (vectors  $[x_1, x_2, \dots, x_M]$  and  $[q_1, q_2, \dots, q_M]$ ). In order to confirm this hypothesis,  $c_{Re}$  was defined as  $c_{Re} = \beta$ . Thus  $c_\lambda$  was considered as function  $\gamma(\alpha, \beta)$  of only two parameters  $\alpha, \beta$  (because  $c_{Re} = \beta$ ):

$$Re^* = (1 - \beta) \cdot \overline{Re}(Q^{in}) + \beta \cdot \overline{Re}(Q^{out}) \quad (9)$$

$$\lambda^* = (1 - \gamma) \cdot \bar{\lambda}(Q^{in}) + \gamma \cdot \bar{\lambda}(Q^{out}) \quad (10)$$

The function  $\gamma(\alpha, \beta)$  was determined using stochastic method. For this purpose, there were generated various values of  $\alpha$  (by assignment of stochastic values of  $Q^{in}$  and  $Q^{out}$ ) and various values of  $\beta$  (by assignment of stochastic  $F(x)$ ), then

corresponding values of  $\gamma$  were calculated. After that there was evaluated dependence of  $\gamma$  on  $\alpha$  and  $\beta$ .

For each generated case, values of  $\lambda_D$  were calculated using the well-known Colebrook-White equation,<sup>[20,21]</sup> which accepted by engineers and scientists as very appropriate or standard of accuracy for the turbulent regime:<sup>[22-24]</sup>

$$\frac{1}{\sqrt{\lambda_D}} = -2 \cdot \log_{10} \left( \frac{\varepsilon}{3.7 \cdot D} + \frac{2.51}{Re \sqrt{\lambda_D}} \right) \quad (11)$$

It is commonly known that Eq. (11) can be calculated for  $\lambda_D$  by iterative methods or using its various explicit approximations such as formulae of Churchill, Swamee, Haaland, etc.<sup>[22-26]</sup>

For identification of  $\gamma$ 's dependence on  $\alpha$  and  $\beta$  there were generated  $10^6$  cases. Each  $k$ -th case includes stochastic parameters of the pipeline (values of  $L_k, D_k, K_k, \varepsilon_k$ ), a cumulative distribution of relative volume of water withdrawals (function  $F_k(x)$ ) and no less than 30 regime data (values of  $T_{k,i}, Q_{k,i}^{in}, Q_{k,i}^{out}$  for each  $i$ -th regime). Each generated quantity had a equiprobable value in the following ranges:  $1 \leq K_k \leq 1.3$ ,  $0.1 \text{ mm} \leq \varepsilon_k \leq 7 \text{ mm}$ ,  $10^2 \text{ m} \leq L_k \leq 10^5 \text{ m}$ ,  $0.2 \text{ m} \leq D_k \leq 1.2 \text{ m}$ ,  $0 < T_{k,i} \leq 150 \text{ }^\circ\text{C}$ ,  $0 < Q_{k,i}^{out} < Q_{k,i}^{in}$ ,  $0 < Q_{k,i}^{in} \leq Q_k^{max}$ . Here,  $Q_k^{max}$  is a maximum possible flow rate for pipeline parameter in the  $k$ -th case. Value of  $Q_k^{max}$  can be determined using the Darcy-Weisbach equation for the  $k$ -th pipeline's length  $L_k$ , the minimum possible Darcy friction factor  $\lambda_{D,k}^{min}$  (that can be calculated by Eq. (11) in case of  $Re \rightarrow \infty$ ) and the maximum possible pressure loss  $\Delta P^{max} = 80 \text{ bar}$  (normally, the water pipe can withstand a maximum pressure of 80 bar):

$$\Delta P^{max} = \lambda_{D,k}^{min} \frac{L_k}{D_k} \frac{(u_k^{max})^2}{2} \rho = \lambda_{D,k}^{min} \frac{8L_k \rho (Q_k^{max})^2}{\pi^2 (D_k)^5}, \lambda_{D,k}^{min} = \left( -2 \cdot \log \left( \frac{\varepsilon_k}{3.7 \cdot D_k} \right) \right)^{-2} \quad (12)$$

After that, for the each  $k$ -th case there was calculated value of  $\beta_k$ , and for the each  $i$ -th regime there was calculated value

of  $\alpha_{k,i}$  by Eq. (4) and value of  $\gamma_{k,i}$ . Calculation of  $\gamma_{k,i}$  consists of the following steps:

- 1) Calculations of  $\overline{Re}(Q_{k,i}^{in})$  and  $\overline{Re}(Q_{k,i}^{out})$  by formula (3);
- 2) Calculation of  $Re_{k,i}^*$  by Eq. (9), using calculated  $\beta_k, \overline{Re}(Q_{k,i}^{in})$  and  $\overline{Re}(Q_{k,i}^{out})$ ;
- 3) Calculation of actual  $\lambda_{D,k,i}^*$  for  $Re_{k,i}^*$  by Eq. (11), using known  $D_k, \varepsilon_k$ ;
- 4) Calculation of  $\lambda_{k,i}^* = (\lambda_{D,k,i}^* + K_k)$ ;
- 5) Calculation of  $\Delta P_{k,i}$ . This value is calculated as sum of  $M + 1$  values of pressure losses on subsections between points  $x_j$  and  $x_{j+1}$  ( $0 \leq j \leq M$ ). Each value of pressure losses on the subsections was calculated by Eq. (2), in which  $l = x_{j+1} - x_j$  and  $u = \frac{4}{\pi D^2} \cdot Q_{k,i}^{in} \cdot (1 - F_k(x_j))$ ;
- 6) Calculations of  $\bar{\lambda}(Q_{k,i}^{in})$  and  $\bar{\lambda}(Q_{k,i}^{out})$  by Eq. (5), using calculated  $\Delta P_{k,i}$ ;
- 7) Calculation of  $\gamma_i$  via Eq. (10), using calculated  $\lambda_{k,i}^*, \bar{\lambda}(Q_{k,i}^{in})$  and  $\bar{\lambda}(Q_{k,i}^{out})$ .

As a result, there were obtained over  $10^7$  points with coordinates  $(\alpha, \beta, \gamma)$ . These points were used in a regression analysis to estimate the relationships between  $(\alpha, \beta)$  and  $\gamma$ . There is evaluated the following dependence of  $\gamma$  on  $\alpha$  and  $\beta$ :

$$\begin{aligned} \gamma(\alpha, \beta) &= \beta \frac{1 - C(\alpha)}{1 - C(\alpha) \cdot \beta}, C(\alpha) \\ &= -0.271 \cdot \alpha^3 - 0.4386 \cdot \alpha^2 + 1.7125 \cdot \alpha \end{aligned} \quad (13)$$

Regarding obtained points  $(\alpha, \beta, \gamma)$ , function  $\gamma(\alpha, \beta)$  has the root mean square error (RMSE) less than 0.02 and the coefficient of determination ( $R^2$ ) more than 0.99. Consequently, other parameters (besides the coefficients  $\alpha, \beta$ ) very slightly affect the coefficient  $c_\lambda$ . Therefore, the suggested hypothesis may be assumed as confirmed and evaluated dependence (9) may be assumed as acceptable for further use. Fig. 5 illustrates obtained points  $(\alpha, \beta, \gamma)$  and functions (9) in various cases of  $\alpha$ . As seen in Fig. 5, if  $\alpha < 0.1$ , then  $\gamma \approx \beta$ .

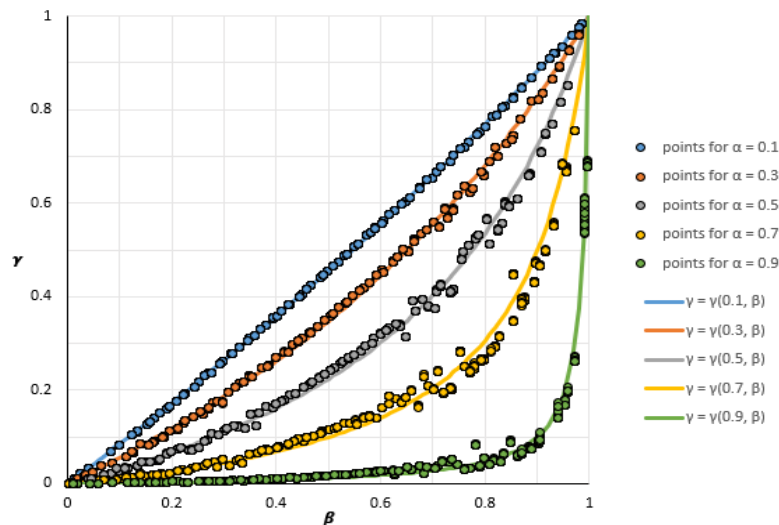


Fig. 5: Example of obtained points  $(\alpha, \beta, \gamma)$  and function  $\gamma(\alpha, \beta)$  in various cases of  $\alpha$ .

$$\left\{ \begin{aligned} Re_i^* &= \frac{4}{v(T_i)\pi D} \cdot ((1 - \beta) \cdot Q_i^{in} + \beta \cdot Q_i^{out}) \\ \lambda_i^* &= (P_i^{in} - P_i^{out} - \rho g(h^{in} - h^{out})) \frac{\pi^2 D^5}{8L\rho} \cdot \frac{\frac{1-\beta}{(Q_i^{in})^2} + \frac{\beta-\beta C}{(Q_i^{out})^2}}{1-\beta C} \\ C &= -0.271 \cdot \left(\frac{Q_i^{in}-Q_i^{out}}{Q_i^{in}}\right)^3 - 0.4386 \cdot \left(\frac{Q_i^{in}-Q_i^{out}}{Q_i^{in}}\right)^2 + 1.7125 \cdot \left(\frac{Q_i^{in}-Q_i^{out}}{Q_i^{in}}\right) \end{aligned} \right. \quad (14)$$

$$\left\{ \begin{aligned} Re_i^* &= \frac{4}{v(T_i)\pi D} \cdot ((1 - \beta) \cdot Q_i^{in} + \beta \cdot Q_i^{out}) \\ \lambda_i^* &= (P_i^{in} - P_i^{out} - \rho g(h^{in} - h^{out})) \frac{\pi^2 D^5}{8L\rho} \cdot \frac{\frac{(1-\beta)(1-C)}{(Q_i^{in})^2} + \frac{\beta}{(Q_i^{out})^2}}{1-\beta C} \\ C &= -0.271 \cdot \left(\frac{Q_i^{out}-Q_i^{in}}{Q_i^{out}}\right)^3 - 0.4386 \cdot \left(\frac{Q_i^{out}-Q_i^{in}}{Q_i^{out}}\right)^2 + 1.7125 \cdot \left(\frac{Q_i^{out}-Q_i^{in}}{Q_i^{out}}\right) \end{aligned} \right. \quad (15)$$

Hence, in accordance with Eqs. (4)-(10) and (13), equivalent points  $(Re_i^*, \lambda_i^*)$  for the function  $\lambda = \lambda(Re)$  in the case of  $Q^{in} > Q^{out}$  can be calculated by the following formula using actual values of  $P_i^{in}$ ,  $P_i^{out}$ ,  $T_i$ ,  $Q_i^{in}$ ,  $Q_i^{out}$ :

### 3.1.3 Case of water injections

Let's consider  $q_j < 0$  ( $1 \leq j \leq M$ ). In this case water flow rate increases along the pipeline section, consequently  $Q^{in} < Q^{out}$ , and branch points are considered as water injection points.

It is commonly known that value of hydraulic pressure losses on a pipeline with constant flow rate along its length does not depend on flow direction. Therefore, the case of water injections can be reduced to the case of water withdrawals by changing the flow direction of each  $j$ -th subsection ( $1 \leq j \leq M + 1$ ) of the pipeline section. Fig. 6 illustrates an example of pipeline section with 3 injection points, in which arrow indicate flow direction, the solid blue lines indicate flows of initial case of water injections, the dotted orange lines indicate flows of tantamount case of water withdrawals, more saturated colors of lines indicate more value of flow rate. As seen in Fig. 6, in the tantamount case absolute values of subsection's flow rates are the same, flow directions are changed only, injection points became withdrawal points.

Thus, in the tantamount case of water withdrawals inlet flow rate is  $Q^{in,*} = Q^{out}$ , outlet flow rate is  $Q^{out,*} = Q^{in}$ , flow rate at the  $j$ -th branch point is  $q_j^* = -q_j$  and, consequently,  $\beta^* = 1 - \beta$ , where initial value of  $\beta$  is calculated by Eq. (6). Using Eq. (14) regarding  $Q^{in,*}$ ,  $Q^{out,*}$ ,  $\beta^*$  and back-substituting, there is obtained a modification of Eq. (14) for the case of water injections:

### 3.2 Identification of the function $\lambda = \lambda(Re)$

The function  $\lambda = \lambda(Re)$  is determined applying  $N$  equivalent points  $(Re_i^*, \lambda_i^*)$  that are calculated by Eq. (14) (in the case of water withdrawals) or Eq. (15) (in the case of water injections) using actual values of  $P_i^{in}$ ,  $P_i^{out}$ ,  $T_i$ ,  $Q_i^{in}$ ,  $Q_i^{out}$ .

If obtained values of  $Re_i^*$  are large enough and values of  $\lambda_i^*$  are nearly identical (water flow is in the fully rough turbulent regime), then, as is known, values of  $\lambda_D$  does not

depend on the Reynolds number and  $\lambda_D$  is constant (consequently  $\lambda$  is constant). Therefore, number of the points  $N$  is not sufficient to identify values of  $\varepsilon$  and  $K$  separately. In this case there can be identified only constant value of generalized friction factor  $\lambda$  for the fully rough turbulent regime:

$$\lambda = \frac{1}{N} \sum_{i=1}^N \lambda_i^* \quad (16)$$

Otherwise, it is possible to identify function  $\lambda = \lambda(Re)$  for the smooth and not fully rough turbulent regimes. For identification of the function  $\lambda = \lambda(Re) = K \cdot \lambda_D(Re, \frac{\varepsilon}{D})$ , it is important to choose accurate formula of the Darcy friction factor which satisfies the following two properties:

**Property 1:** There can be directly determined coefficients  $K$  and  $\varepsilon$  so that the function  $\lambda(Re) = K \cdot \lambda_D(Re, \frac{\varepsilon}{D})$  is close to the given  $N$  points  $(Re_i, \lambda_i)$ .

If selected function  $\lambda_D = \lambda_D(Re, \frac{\varepsilon}{D})$  in Eq. (1) satisfies the property 1, then coefficients  $K$  and  $\varepsilon$  can be directly calculated by points  $(Re_i, \lambda_i)$  avoiding any complex iterative methods for the searching values of  $K$  and  $\varepsilon$ .

**Property 2:** For any values of  $\varepsilon/D$ ,  $Re$  and positive number  $k$ , there can be determined functions  $f_1(k)$  and  $f_2(k, \frac{\varepsilon}{D})$  which provide the following Eq. (16):

$$\lambda_D(k \cdot Re, \frac{\varepsilon}{D}) = f_1(k) \cdot \lambda_D(Re, f_2(k, \frac{\varepsilon}{D})) \quad (17)$$

In practice, the diameter of water flow cross-section may differ from the given pipe's inner diameter  $D$ . In this case, if the selected function  $\lambda_D = \lambda_D(Re, \frac{\varepsilon}{D})$  satisfies property 2, then the pressure losses can be defined by the same Eq. (2) with given  $D$  and the imprecision of  $D$  in leads only to «mutation» of coefficients  $K$ ,  $\varepsilon$  in Eq. (2).

Let  $D_r$  is the diameter of flow area,  $K_r$  is the minor losses coefficient, and  $\varepsilon_r$  is the roughness of inner wall of the pipe or the deposition surface. Values of flow velocity  $u$  and the Reynold's number  $Re$  (which calculated by measured flow rates using given diameter  $D$ ) are related to real flow velocity  $u_r$  and the real Reynold's number  $Re_r$  as follows:  $\frac{u_r}{u} = (\frac{D_r}{D})^{-2}$  and  $\frac{Re_r}{Re} = (\frac{D_r}{D})^{-1}$ . Substituting  $D_r$ ,  $K_r$ ,  $\varepsilon_r$  and relations

for  $u_r, Re_r$  to Eqs. (17) and (2) gives the same equality Eq. (2) for pressure losses but with «mutated» coefficients  $K, \varepsilon$  using Eq. (18):

$$K = K_r \cdot \left(\frac{D_r}{D}\right)^{-5} \cdot f_1\left(\frac{D}{D_r}\right), \quad \varepsilon = D \cdot f_2\left(\frac{D}{D_r}, \frac{\varepsilon_r}{D_r}\right) \quad (18)$$

Two above-described properties are satisfied by the well-known Alshul formula  $\lambda_D = 0.11 \left(\frac{68}{Re} + \frac{\varepsilon}{D}\right)^{0.25}$  and formula  $\lambda_D = 0.3164 \left(\frac{1}{Re^{0.83}} + 0.11 \cdot \frac{\varepsilon}{D}\right)^{0.3}$  proposed by Chen.<sup>[27]</sup> However, for  $4000 \leq Re \leq 10^8$  and  $0.0001 \leq \frac{\varepsilon}{D} \leq 0.01$  these formulae demonstrate insufficient accuracy: maximum relative errors are more than 9 %.<sup>[22]</sup> Other approximations based on the power-low formulation of  $Re$  such as approximations of Moody and Wood don't satisfy the property 2 and also demonstrates insufficient accuracy.<sup>[24]</sup> Approximations based on the logarithmic formulation of  $Re$  (such as approximations of Haaland, Swamee-Jain, Zigrang-Sylvester, Manadilli, Fang, which demonstrate more sufficient accuracy,<sup>[22,24]</sup> don't satisfy the property 2. Also, it is unclear how these approximations can satisfy the property 1. Logarithmic formulation of  $Re$  in these approximations makes impossible to define  $f_1$  and  $f_2$  in Eq. (17) and significantly complexifies determination of  $K, \varepsilon$  for the function  $\lambda(Re) = K \cdot \lambda_D(Re, \frac{\varepsilon}{D})$ . Thus, in purpose to effectively identify the function  $\lambda = \lambda(Re) = K \cdot \lambda_D(Re, \frac{\varepsilon}{D})$  by known points  $(Re_i, \lambda_i)$  in case of the probable depositions on pipe's inner wall, there was suggested a new approximation of the Colebrook-White equation:

$$\lambda_D(Re, \frac{\varepsilon}{D}) = 0.1212 \cdot \left(\frac{32.3532}{Re^{0.852}} + \left(\log_{0.1} \frac{\varepsilon}{D}\right)^{-5.2785}\right)^{0.31508} \quad (19)$$

The suggested approximation Eq. (19) of the Darcy friction factor has maximum relative error 1.92 % at the range  $4000 \leq Re \leq 10^8$  and  $0.0001 \leq \frac{\varepsilon}{D} \leq 0.01$  (comparing to friction factor's values obtained by the Colebrook-White equation). In comparison, on this range the maximum relative errors of the Haaland's and Altshul's approximations are 1.4% and 9.1%, respectively.<sup>[22]</sup> Hence, the suggested Eq. (19) may be assumed as acceptable for further use. Fig. 7 demonstrates similarity of friction factors which obtained by the solution of the Colebrook-White equation and using the suggested approximation.

The suggested Eq. (19) satisfies the property 1. It is possible to determine  $K$  and  $\varepsilon$  by minimizing the following residuals sum:  $\sum_{i=1}^N \left(\lambda_i^{\frac{1}{0.31508}} - \left(\lambda(Re_i)\right)^{\frac{1}{0.31508}}\right)^2 \rightarrow \min$ .

Expressions  $\lambda_i = K \cdot \lambda_D(Re_i, \varepsilon)$  with Eqs. (20)-(22) can be reduced to the linear regression of the form  $y_i = Ax_i + B$  by the following substitutions:  $y_i = \frac{1}{0.1212} \lambda_i^{\frac{1}{0.31508}}, x_i = \frac{32.3532}{Re^{0.852}}$ ,  $A = K^{\frac{1}{0.31508}}, B = K^{\frac{1}{0.31508}} \cdot \left(\log_{0.1} \frac{\varepsilon}{D}\right)^{-5.2785}$ . Using the least-squares method, calculation of the coefficients  $A$  and  $B$  of the linear regression can be reduced to simple solution of two linear equations for  $A$  and  $B$ . Using determination of  $A$  and  $B$  (regarding  $x_i, y_i$ ), back-substituting gives expressions for  $K$  and  $\varepsilon$ :

$$K = \left(\frac{N \cdot S_{xy} - S_y \cdot S_x}{N \cdot S_{xx} - S_x \cdot S_x}\right)^{0.318508}, \quad \varepsilon = D \cdot 0.1 \left(\frac{N \cdot S_{xy} - S_y \cdot S_x}{S_y \cdot S_{xx} - S_x \cdot S_{xy}}\right)^{\frac{1}{5.2785}} \quad (20)$$

$$S_y = \sum_{i=1}^N \frac{1}{0.1212} (\lambda_i^*)^{\frac{1}{0.31508}}, \quad S_x = \sum_{i=1}^N \frac{32.3532}{(Re_i)^{0.852}} \quad (21)$$

$$S_{xy} = \sum_{i=1}^N \frac{266.9406}{(Re_i)^{0.852}} (\lambda_i^*)^{\frac{1}{0.31508}}, \quad S_{xx} = \sum_{i=1}^N \left(\frac{32.3532}{(Re_i)^{0.852}}\right)^2 \quad (22)$$

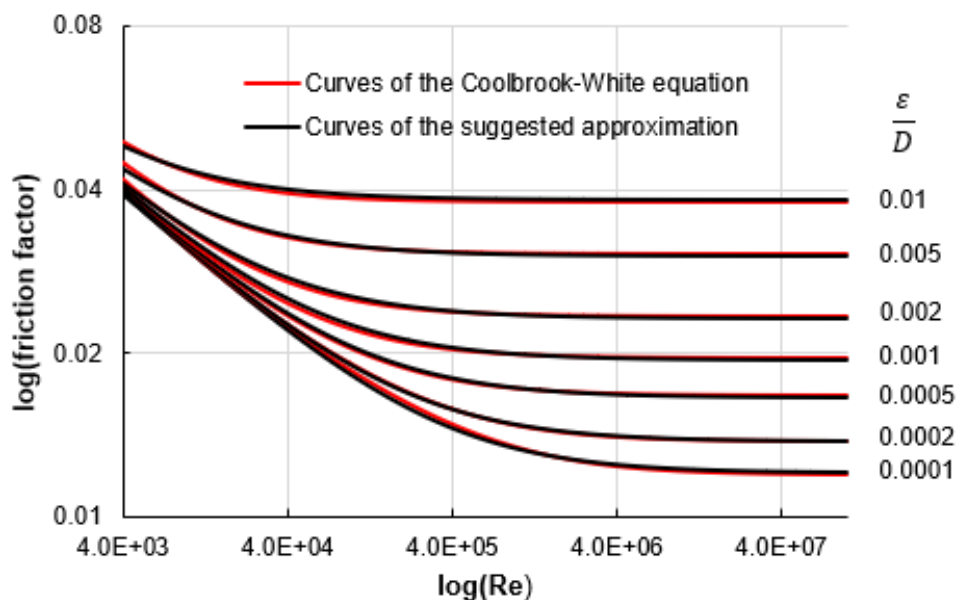


Fig. 7: Comparison of the Colebrook-White equation and the suggested approximation.

The suggested Eq. (19) satisfies the property 2. Substituting  $k \cdot Re$  as argument to the approximation (Eq. (19)) gives definitions of the functions of the property 2:  $f_1(k) = k^{-0.268448}$  and  $f_2\left(k, \frac{\varepsilon}{D}\right) = \left(\frac{\varepsilon}{D}\right)^{k^{-0.1614095}}$ . In accordance with Eq. (18) these functions determine dependences of «mutated» coefficients  $K$ ,  $\varepsilon$  on real values of  $K_r$ ,  $\varepsilon_r$  and  $D_r$ :

$$K = K_r \left(\frac{D_r}{D}\right)^{-4.731552}, \quad \varepsilon = D \cdot \left(\frac{\varepsilon_r}{D_r}\right)^{\left(\frac{D_r}{D}\right)^{0.1614095}} \quad (23)$$

Eq. (23) for  $K$  allows to estimate thickness of the deposition on pipe’s inner wall (values from  $t_{min}$  to  $t_{max}$ ) using calculated value  $K$  and assumption that typically real minor loses are no more than 5% of linear pressure losses. Using inequality  $1 \leq K_r \leq \min(1.05, K)$ , let’s define the range of the thickness’s value:

$$t_{min} = \frac{1}{2}D \left(1 - \left(\frac{\min(1.05, K)}{K}\right)^{0.211347}\right), \quad t_{max} = \frac{1}{2}D \left(1 - \left(\frac{1}{K}\right)^{0.211347}\right) \quad (24)$$

Hence, function  $\lambda = \lambda(Re)$  of the form (17) are identified using calculation of its coefficients  $\varepsilon$  and  $K$  by Eq. (18). A comparative analysis of the proposed model in relation to previously published approaches is presented in Table 1. This table highlights the key differences regarding complexity and applicability.

#### 4. Comparison with actual data

There was considered a section (449 km – 547 km) of the Astrahan-Mangyshlak trunk water pipeline located in western Kazakhstan. Pipe’s inner diameter is 1.196 m. The pipeline section contains 6 water withdrawal points, which located in 490 km, 503 km, 515 km, 530 km, 545.7 km and 546 km. Inlet and outlet points of the pipeline section have sensors of pressure, temperature and water flow rates.

There was used information on water consumers of the pipeline. The coefficient  $\beta$  was 0.701. It was calculated using information about volume of consumers’ monthly water usage and their withdrawal location at pipeline. Resistance coefficients and dependence  $\lambda = \lambda(Re)$  for the pipeline section were identified using the sensors data for normal modes with water withdrawals (382 values of  $P_i^{in}, P_i^{out}, T_i, Q_i^{in}, Q_i^{out}$ ). Resistance coefficients and dependence  $\lambda = \lambda(Re)$  were validated using the sensors data for specific modes without withdrawals (5 values of  $P_i^{in}, P_i^{out}, T_i, Q_i$ ).

Using parameters of the pipeline sections and sensors data for its normal modes  $P_i^{in}, P_i^{out}, T_i, Q_i^{in}, Q_i^{out}$ , there were calculated two points’ group:  $(\overline{Re}(Q_i^{in}), \bar{\lambda}(Q_i^{in}))$  and  $(\overline{Re}(Q_i^{out}), \bar{\lambda}(Q_i^{out}))$ . These points’ group are indicated on Fig. 8 by blue and green colors respectively. After that, there were calculated equivalent points  $(Re_i^*, \lambda_i^*)$  by the above-described formulae (see fig.8, orange points). All these points are shown in fig. 8 using semitransparent colors with the aim of different display of plot areas with high and low density of points. Then there was determined resistance coefficients and function  $\lambda = \lambda(Re)$  using the above-described approach (see Fig. 8, black curve). After that, there were calculated actual points  $(\overline{Re}(Q_i), \bar{\lambda}(Q_i))$  for the specific modes without withdrawals (case  $Q_i^{in} = Q_i^{out} = Q$ ). Fig. 8 indicates points  $(\overline{Re}(Q_i), \bar{\lambda}(Q_i))$  by black color.

As seen in Fig. 8, determined function  $\lambda = \lambda(Re)$  for the pipeline section demonstrates a good agreement with actual points  $(\overline{Re}(Q_i), \bar{\lambda}(Q_i))$ . Hence, the above example can serve as a confirmation of the correctness of usage of the above-described approach to determine actual function  $\lambda = \lambda(Re)$  in case of intermediate withdrawals using measurements at the pipeline sections’ ends.

**Table 1:** Comparison of the proposed model with previous models.

Feature	Models [11-14]	Models [15-17]	Proposed Model
Consideration of branching	Not considered	Considered	Considered
Computational complexity	Simple calculations	Requires solving nonlinear equations	Simple calculations
Result evaluation (R <sup>2</sup> , MSE)		Not supported (errors are not obvious and may significantly distort results)	Supported
Noise/error filtering in input data ML applicability	Supported		
Need for information on withdrawal/injection distribution	Does not consider withdrawals/injections at all	Not required	Required (but typically known in practice)
Consideration of the effect of internal diameter reduction (deposition) on $\lambda = \lambda(Re)$ in the smooth and transitional turbulent flow regimes.	Not considered (incorrect $\varepsilon$ and $\lambda(Re)$ if $D_r < D$ )		Allows estimation of $D_r$ and $\varepsilon_r$ (assesses pipe siltation/deposition)

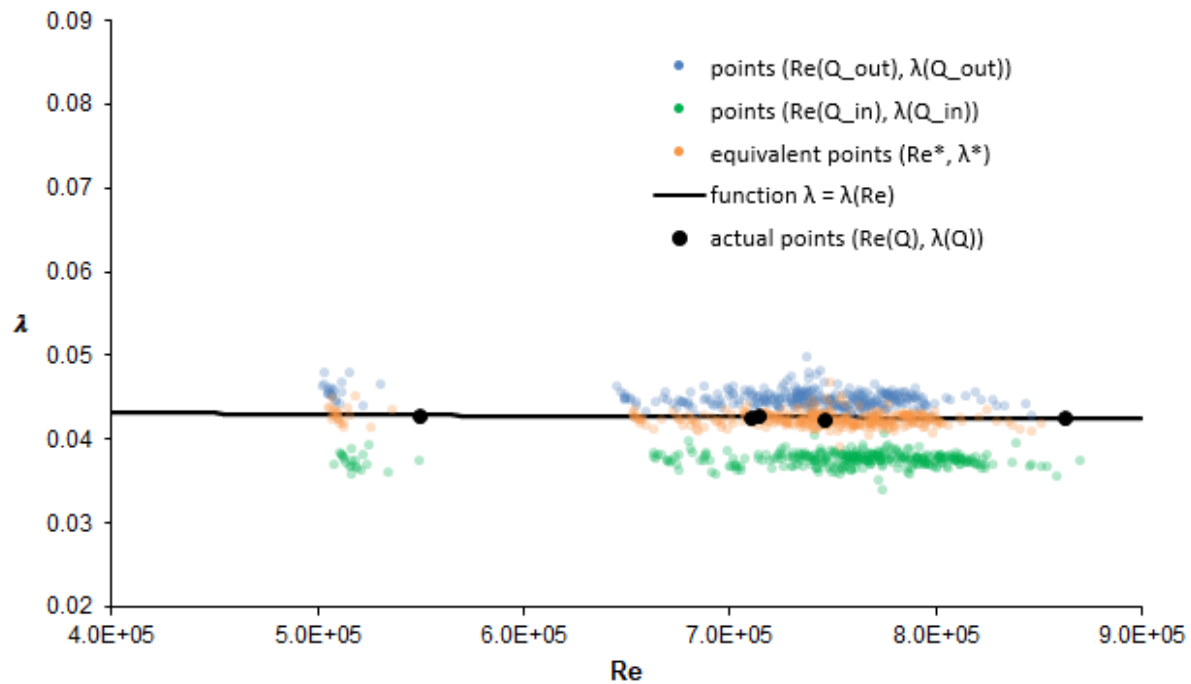


Fig. 8: Function  $\lambda = \lambda(Re)$  and points used for its identification and validation.

## 5. Discussion

1) The above-described approach considers the enforced (pumped) flow along the entire length of the pipeline section. If elevation profile of the pipeline section contains points that noticeably higher than heights of inlet and outlet points, there may be subsections with gravity flow. Eqs. (2) and (3) are not suitable for the case of gravity flow and thus Eqs. (5), (15), and (16) give invalid equivalent points for the function  $\lambda = \lambda(Re)$ .

2) In practice, sensor's readings of pressure, temperature and water flow rate may have significant measurement errors. These errors are caused by the sensors and by the fact that each reading can be measured at a slightly different time moment. For the second cause measurement errors are more significant in case of non-stationary pumping mode that changes dramatically. Also, in practice, the distribution of water withdrawals volume and, thus, the coefficient  $\beta$  can be not constant over time. Hence, there is additional analysis required to evaluate the sensitivity of identification of function  $\lambda = \lambda(Re)$  to measurement errors of  $P_i^{in}$ ,  $P_i^{out}$ ,  $T_i$ ,  $Q_i^{in}$ ,  $Q_i^{out}$  or to time-to-time variability of the coefficient  $\beta$ . On the other hand, it is evident that if measurement errors of  $P_i^{in}$ ,  $P_i^{out}$ ,  $T_i$ ,  $Q_i^{in}$ ,  $Q_i^{out}$  has dominant the random error's part (the systematic error's part is minimal) and assumed value of  $\beta$  is the average over time, then increasing the number of measurements leads to decreasing the error of identification of the function  $\lambda = \lambda(Re)$ .

3) The above-described approach may be modified for cases of other types of fluids. For instance, oil (or gas) pipelines also contain withdrawals and injection points, and measurements of pressure, temperature and flow rate may not occur in all branch points. In these cases, there should be

considered also laminar and transient regimes of flow, and, perhaps, thermal conduction of pumped fluid with the environment.

4) The suggested method is especially important for low- and middle-income countries (LMICs), where limited measurement tools and funding frequently impede precise monitoring and modeling of water pipeline networks. This strategy minimizes the costs linked to identifying network parameters by necessitating data solely from the endpoints of a pipeline section.

This can help in optimizing water distribution management, minimizing losses, and enhancing service quality in resource-limited infrastructure environments.

5) The paper approach has disadvantages and advantages comparing to these complex models.

Disadvantages: the paper approach considers a narrower formulation (colocations of pressure and flow rate sensors) and requires information on distribution of withdrawal or injection volumes.

Advantages:

- The paper approach requires much simpler calculations.
- Proposed method allows to determine the friction factor as a function of the Reynolds number rather a constant value.
- In practice, readings of sensors contain measurement errors. The approach allows to calculate various criteria of result's accuracy (such as MSE, R2) using regression analysis of the «equivalent» points.
- Considers the narrowing of the actual pipe diameter (due to siltation and fouling) and allows for an estimation of the decrease in diameter.
- This approach is well-suited for machine learning applications, especially during the preprocessing stage.

## 6. Conclusion

This paper describes steps of resistance coefficients' formulae derivation for sections of water pipeline network in case of intermediate water withdrawals or injections using measurements only at inlet and outlet points of the section. There was proposed an additional coefficient  $\beta$  to characterize variety of all intermediate branches' parameters such as its locations and relative volumes of water withdrawals or injections. After that, using coefficient  $\beta$  and applying stochastic method and regression analysis, there were obtained formulae for determination of «equivalent» points for actual dependency of generalized friction factor on the Reynolds numbers for the pipeline segments using measurements of the segment's end. Procedures for obtaining the formulae are described in detail. Hence, these formulae allow to identify the resistance parameters of the pipeline sections in relatively simple way (without solution of complex systems of nonlinear equations). The paper contains an example of using this approach on real water pipeline with multiple withdrawals. The example may serve as a confirmation of the correctness of usage of the proposed approach. There may be further research to modify the proposed approach for the case of gravity flow or for oil and gas pipeline network. The proposed approach offers computational efficiency through simplified calculations, enables friction factor estimation via Reynolds-number dependence, accommodates measurement uncertainties through regression-based accuracy metrics, and accounts for practical pipe diameter reductions.

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## Conflict of Interest

There is no conflict of interest.

## Supporting Information

Not applicable.

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