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Sustainability Analysis of Energy Resources Transport Based on A Digital N-D Logistics Network

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Abstract

This study employs an analytical approach to scrutinize the dynamics of network streaming phenomena, specifically the transfer processes of continuous and discrete traffic flows across n-dimensional network structures. Introducing an evolutionary differential system in a generalized framework, we establish a summable function as a generalized solution that satisfies a crucial integral identity. This reliance on generalized solutions enhances the precision in depicting the physical nature of transported flows and elucidates the study of dynamic processes within multidimensional network-like domains. Employing a mathematical model, we apply it to n-dimensional flows with distributed parameters in network models. Our approach leverages generalized solutions and the construction of a compact family of approximations within the chosen state space. The results shed light on fundamental challenges related to optimal control and stabilization of differential systems, encompassing those with delays. Furthermore, the study unveils optimal and sustainable energy delivery strategies for both short and long-term scenarios. Notably, utilizing a digital N-D logistics network with real-world data facilitates a thorough assessment of the environmental sustainability implications of energy resource transport operations. These findings underscore the system's efficacy in guiding policymakers toward formulating sustainable energy policies for a greener future.

Keywords: Digital logistics; Network-like multidimensional region; Flows of energy resources; Generalized solution-D networking.

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1. Introduction

Network effects have become a key driver of value creation as more products and services have become digital.[1] A digital business strategy differs from traditional information technology (IT) strategies, transcending traditional functional areas, such as marketing, logistics, and operations.[2] Digital technologies are changing supply-chain business processes based on artificial intelligence (AI)/machine learning (ML) and blockchain.[3] Digital logistics platforms integrate blockchain and Internet of Things (IoT) technologies into modern supply chains.[4] Digital devices transform traditional marketing into digital forms,[5] using interactive marketing, such as social media, television, radio channels, SMS, email, search engines, websites, mobile apps, electronic billboards, and social networks.[6] Data intelligence allows the improvement of digital marketing.[7] Three organizations' forms of digital platforms (vertical, horizontal, and modular), generating different degrees of complementary engagement, were identified. There is a need for more results for the mathematical sustainability theory, focused on evolutionary differential equations and systems with distributed parameters in network-like structures.[8] In this regard, the authors paid attention to a series of works,[9-11] that present mathematical models with a system of online-to-offline (O2O) channels. The analogy of power transmission in an online-to-offline (O2O) supply-chain management network model of a small

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manufacturer of household electrical components is used to deliver goods to retail centers and retailers.[9]

It has explored collaborative pricing and inventory policy for a tiered closed-loop supply chain (C.L.S.C.) based on a recovery model with an O2O sales channel.[10] The sustainability problem included a conversation about network structure.[11] Emphasized supply-chain model with one supplier and one buyer for one type of product to improve the quality of service.[11] To ensure improvement in the quality of service, the model used the concept of «online-to-offline» (O2O). To increase the future profitability of the system, a particular customer acquisition cost for the O2O channel is considered. In the model, as in the electricity transmission and distribution network, the firm's O2O target market is available to retailers in certain cities. The main distribution center is located in the vicinity of the city. Production facilities are located at a considerable distance from the city. The company's fleet of trucks delivers goods to the main distribution center. Delivery to various areas of the city, where retailers are located, is carried out by small-sized vehicles belonging to the company. The production capacity of each facility is considered to be the electricity generation capacity.

The distance between nodes stands for the length of power transmission wires. The amount of product transferred from one node to another is the amount of transmitted power. Retail store demand is per customer cost load. To illustrate the practical application of the model, an experimental study was carried out with four power plants, three distribution centers and five consumer zones of 50 consumers each. The results of a numerical experiment of mixed integer linear programming confirmed the successful satisfaction of the retail demand of each consumer zone. The minimization of the total cost of the supply chain has also been proven.

We explored collaborative inventory and pricing solutions within multi-layered closed-loop supply chain management with vehicle recovery and return (R.T.I.).[10] The presented model of management and restoration of a closed supply chain (C.L.S.C.) focuses on O2O.[9] The manufacturer's market demand is met through many retailers, each opening an online sales channel. The manufacturer has two types of products. The first type is made from raw materials, and the second - is from already-used products collected from consumers. Finished products are transported in reusable secondary packaging owned by a third-party logistics provider (3PL). 3PL delivers the assembled used products from the end user to the manufacturer for further recovery. To model a centralized system, the total supply chain revenue is calculated, followed by the cost of the individual player and then the net profit. The results showed that investments in the O2O channel lead to a significant increase in profits. It has also been proven that, in contrast to traditional modeling, total profit increases with an increase in the percentage of defective products when disposing of defective products exceeds the cost of outsourcing.[11] If the cost of outsourcing is higher than the cost of scrapping, the model converges to traditional models in which profit decreases as scrap increases. A single supplier, single buyer supply chain management system with a single product type (S.C.M.) is considered when there is an unreliable supplier and an O2O environment. In the study, the overall system profit was optimized along with optimized values for shipping volume, safety factor, lead time, selling price, quality, service, order cost, setup cost, and the probability of going into an "out of control" state. Sett *et al.* demonstrated that the model improves product quality and fixed service.

Considering the extension of the proposed approach, this study suggests implementing the n-D methodology based on digital logistics networks. The research proposed considering that network interactions with customers become more complex digital networks assessed in several dimensions. In this sense, the authors suggested the term "network-like multidimensional structures" concerning the complex vision of omnichannel interactions.[12,13] based on the integration of logistics and marketing. This research aims to develop a mathematical apparatus for investigating such multidimensional interactions. The investigation started with a deep analysis of the sustainability of natural processes for developing the n-D methodology. In numerous applications, it is through such mathematical formalisms that natural processes and phenomena can be described with a sufficient degree of adequacy.[14,15] One feature in analyzing such processes is the complex rheology of their carriers, which entails a significant expansion of the class of generalized solutions and the use of abstract function spaces for mathematical descriptions of network phenomena. The choice of these spaces is determined by the requirement to preserve the theorems of existence and uniqueness with an arbitrary interval of change for a time variable if this corresponds to the nature of the phenomenon or process under study.[16,17] The authors provide a rather detailed proof of the sustainability of a generalized solution of a differential system to demonstrate an analogue of the Lyapunov sustainability concept of the undisturbed state of a differential system, as applied to evolutionary systems with partial derivatives and to show the possibility of using the previously obtained results in the case under study.[18]

The approach used to analyze the process under study, based on the reduction of a differential system to a differential-

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difference system, indicates the path of algorithmization, and hence subsequent digitalization, in the direction of forming a complex of problem-oriented computer programs that are adapted to the tasks of applied analysis, determined by the logistics of processes in network networks—multidimensional structures. The authors are attempting to reach the three primary purposes in the present study.

Firstly, the concept of a network-like region of change of a spatial variable and the principles of constructing a mathematical model for analyzing the stability property of the process of transporting continuous media over a network multidimensional carrier are explained in sufficient detail, as well as the necessary Sobolev-type spaces for studying the initial-boundary value problem (differential system) of this model. Secondly, using the reduction of a differential system to a differential-difference system, conditions are given for the solvability of a differential system in the class of summable functions. Finally, the concept of stability of a solution of a differential system is defined, similar to the classical one in the sense of Lyapunov, and conditions guaranteeing this stability are indicated. The main critical contributions of the paper include:

• Improved network performance: Sustainable analysis of energy resources transport provides an opportunity for better network performance by allowing detailed, dynamic, and accurate tracking of energy resources, facilitating better use of available resources and reducing network wastage.

• Reduced emissions and improved energy efficiency: Accurate tracking of energy resources means there is a higher likelihood that their transport is being carried out most efficiently, thus reducing the carbon emissions associated with their transport.

• Enhanced supply chain visibility: Digital N-D logistics networks provide extensive visibility into the supply chain, thus enabling better decision-making regarding the transport of energy resources.

• Improved billing accuracy: Digital N-D logistics networks reduce the chances of billing inaccuracy, as each transport process is tracked in real-time, and all information is available for analysis.

• Increased savings: Real-time analysis of energy resources transport data through digital N-D logistics networks can help reduce costs associated with transport, such as fuel costs, thus leading to overall savings.

• Improved customer experience: By providing accurate and timely information about energy resources transport, customers are more likely to have a better experience and trust the provider.

This study aims to establish the conditions that guarantee the stable transfer of a continuous medium over a network transport carrier. For this, a mathematical model of the network carrier of the process is built (Section 2.1 Mathematical Model description and definitions), and further (Section 3. Results and Discussion), a mathematical model of the network process is formed and substantiated. In contrast (Section 3.2. Sustainability of the Differential System), a detailed description of various types of network media is illustrated in detail (graphical interpretation).

The primary mathematical analysis tool (along with justification) is introduced differently -different models approximating the original. The main result with its explanation is shown in the final part of Section 3.2. Sustainability of the Differential System. The study ends (4. Discussion, 5. Conclusions) with a description of the results obtained when used in various areas of natural Science.

2. Materials and methods

The authors illustrate the flow chart diagram in Fig. 1. The process starts with initial data acquisition from suppliers, customers, and the environment.

2.1 Mathematical model

Focusing on the description of a carrier in the process of transporting a continuous medium as the Nomenclature can be read in Appendix 1. A network-like region, $\Im$ which is a set of Euclidean space $\mathbb{R}^n$s, $n \geq 2$. The network-like structure of the region $\Im$ determines the presence of sub-regions $\Im_l$ ($l = \overline{1, N}$) in its composition, which are interconnected (adjacent to each other) in certain places, ω_j ($j = \overline{1, M}$, $1 \leq M \leq N-1$), of the region $\Im$ through parts of their boundaries, $\partial\Im_l$ ($l = \overline{1, N}$):

$\Im = \hat{\Im} \cup \hat{\omega}$ $\hat{\Im} = \bigcup_{l=1}^{N} \Im_l$, $\hat{\omega} = \bigcup_{j=1}^{M} \omega_j$ $\Im_l \cap \Im_j = \emptyset$ $(l \neq j)$, $\omega_l \cap \omega_j = \emptyset$ $(l \neq j)$, $\Im_l \cap \omega_j = \emptyset$ $\forall l \neq j$.[19,20]

From now on, we will refer to such junctions ω_j as "nodal", and they are defined by surfaces S_j (meas $S_j > 0$), consisting of a union of surfaces S_{ji} (meas$S_{ji} > 0$) $i = \overline{1, m_j}$

$S_j = \bigcup_{i=1}^{m_j} S_{ji}$ $S_j \subset \partial\Im_{l_0}$ $S_{ji} \subset \partial\Im_{l_i}$ surfaces S_{ji} are assumed to be smooth, surfaces S_j are assumed to be piecewise-smooth $i = \overline{1, m_j} j = \overline{1, M}$,

Note that $n = 3$ a network-like region $\Im$ is a complex industrial structure for transporting energy carriers of various types (oil and gas pipelines, water lines, heat lines, *etc*).

The mathematical description of transporting a continuous medium is preceded by introducing necessary normed function spaces with certain generalized derivatives; the choice of spaces is limited only by the requirement to preserve the theorem of existence.[21]

This study uses classical spaces $L_2(\Omega)$ $W_2^1(\Omega)$ $(\Omega \subset \mathbb{R}^n)$, their representations of scalar products, and norms. It also uses the classical Lebesgue integral for summable functions in a network-like structure, as shown in Equation 1.

$$\int_{\Im} u(x)\, dx = \sum_{l=1}^{N} \int_{\Im_l} u(x)\, dx \qquad (1)$$

Based on the representation of the region, we obtain the following representations of the classical spaces in Equation 2.

$L_2(\Im)$ $W_2^1(\Im)$, $L_2(\Im) = \prod_{l=1}^{N} L_2(\Im_l)$, $W_2^1(\Im) = \prod_{l=1}^{N} W_2^1(\Im_l)$, where,

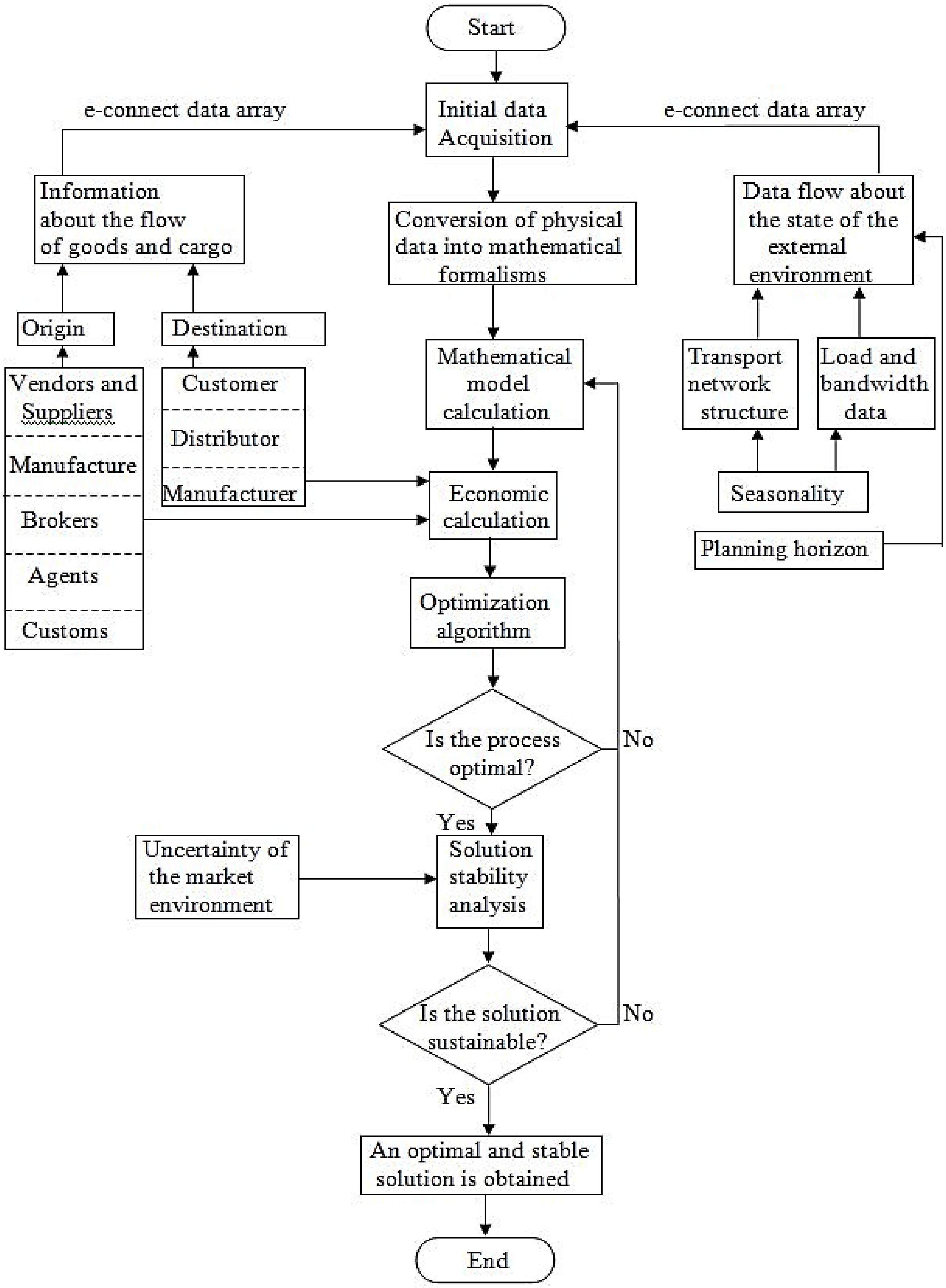

Fig. 1 The flow chart diagram.

$$PuP_{L_2(\mathfrak{I})} = (\sum_{l=1}^{N} \int_{\mathfrak{I}_l} u^2(x)\, dx)^{1/2},$$

$$PuP_{W_2^1(\mathfrak{I})} = (\sum_{l=1}^{N} \int_{\mathfrak{I}_l} [u^2(x) + \sum_{\kappa=1}^{n} (\frac{\partial u(x)}{\partial x_\kappa})^2] dx)^{1/2} \quad (2)$$

The first step in describing a mathematical model of transporting a continuous medium is carried out by constructing a particular class of functions $u(x)$ with spatial variable $x = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n$ s, determining the bare admissible functional spaces for these models. Traditionally denoting the variety of functions, $u(x)$ which are continuous on closure $\overline{\Omega}$ $C(\overline{\Omega})$, we introduce a variety $C^1(\overline{\Omega})$ of differentiable functions $u(x)$ and derivatives $u_{x_\kappa}(x)$, $\kappa = 1,2,\ldots,n$ of which the boundary $\partial\Omega$ is defined by the extension $u_{x_\kappa}(x)$ along the continuity Ω, too.

$C^1(\overline{\Omega})$ classical relations for differentiable functions define the norm and the scalar product. As a result, we obtain

$C^1(\overline{\Im}) = \prod_{l=1}^{N} C^1(\overline{\Im}_l)\ \overline{C} = \overline{\Im}$

From Variety $C^1(\overline{\Im})$, single out a collection of functions $\tilde{C}^1(\overline{\Im})$ consisting of elements $u(x) \in C^1(\overline{\Im})$ which satisfy the following conditions for S_j: S_{ji} $(i = \overline{1, m_j})$:

$\int_{S_j} a(x)\frac{\partial u(x)}{\partial \bar{n}_j} ds + \sum_{i=1}^{m_j} \int_{S_{ji}} a(x)\frac{\partial u(x)}{\partial \bar{n}_{ji}} ds = 0 \forall \omega_j, j = \overline{1, M},$ (3)

Where equation (3) is the condition of adjoining $\Im_l$ to each other.[18,19] Functions $a(x) \in L_2(\Im)$ and derivatives $\frac{\partial u(x)}{\partial \bar{n}_j}$, $\frac{\partial u(x)}{\partial \bar{n}_{ji}}$ relation (3) should be understood as their parts, defined as $S_j S_{ji}$ and, and vectors $\bar{n}_j$ and $\bar{n}_{ji}$ define the external normals for $S_j S_{ji}$ and $i = \overline{1, m_j} j = \overline{1, M}$.

The following two definitions introduce the admissible functional spaces essential for mathematical models transporting a continuous medium.

Definition 1. Space $\widetilde{W}^1(\Im)$ is the closure of collection $\tilde{C}^1(\overline{\Im})$ concerning the norm $P \cdot P_{W_2^1(\Im)}$ of space $W_2^1(\Im)$.

Assuming that functions $u(x) \in \tilde{C}^1(\overline{\Im})$ satisfy boundary conditions $u(x)|_{x\in\partial\Im} = 0$, we obtain a reduction $\tilde{C}_0^1(\overline{\Im})$ of space $\tilde{C}^1(\overline{\Im})$.

Definition 2. $\widetilde{W}_0^1(\Im)$ The closure of reduction $\tilde{C}_0^1(\overline{\Im})$ concerning the norm $P \cdot P_{W_2^1(\Im)}$ of space $W_2^1(\Im)$ defines space. Introduced spaces $\widetilde{W}_0^1(\Im)$ $\widetilde{W}^1(\Im)$ are used as the spaces of admissible solutions for Dirichlet boundary value problems and more general boundary value problems.

Note 1. Note the feature of space $\widetilde{W}^1(\Im)$: if an element $u(x) \in \widetilde{W}^1(\Im)$, then its reduction by $\Im_l$ for each fixed l $(l = \overline{1, N})$ is an element of $W^1(\Im_l)$, and then generalized derivatives $u_{x_\iota}(x)$ exist $\Im_l$. Hence, all functions from $\widetilde{W}^1(\Im)$ condition (1) are described as adjacency conditions $\omega_j j = \overline{1, M}$.

Note 2. Embedding $\widetilde{W}_0^1(\Im) \subset \widetilde{W}^1(\Im)$ $\widetilde{W}_0^1(\Im)$ and, according to Definition 1, is a separable space; moreover, if a sequence belonging to $\widetilde{W}_0^1(\Im)$ converges weakly, then its boundary element also belongs to $\widetilde{W}_0^1(\Im)$. Table 1 describes the mathematical expression.

Below, we consider an initial boundary value problem underlying the mathematical models of transporting a continuous medium. Spaces $\widetilde{W}_0^1(\Im)$ $\widetilde{W}^1(\Im)$ obtained above contain traces of functions $u(x,t)$ defined in the region $\Im_T = \Im \times (0,T)$ $T < \infty$; similar to Lebesgue $L_2(\Im)$ and Sobolev $W_2^1(\Im)$ spaces introduced above, spaces $L_p(\Im_T)$ $(p = 1,2)$ $W_2^{1,0}(\Im_T)$ $W_2^1(\Im_T)$ are used with classical scalar products and norms for elements.

Definition 3.

Closure in the norm $W_2^{1,0}(\Im_T)$ of a collection of functions, $y(x,t) \in L_1(\Im_T)$ summable in the region $\Im_T$, having traces of $y(x,t_0) \in \widetilde{W}_0^1(\Im)$ $t_0 \in (0,T)$ it, continuously depending on a variable $t \in (0,T)$ in the norm $W_2^1(\Im)$, is called space $\widetilde{W}_0^{1,0}(\Im_T)$.

Definition 4.

If the closure of the collection of summable functions $y(x,t)$ specified in Definition 4 is carried out with the norm $W_2^1(\Im_T)$, we obtain space $\widetilde{W}_0^1(\Im_T)$.

It is easy to demonstrate that $\widetilde{W}_0^1(\Im_T)$ it is tightly embedded in space $\widetilde{W}_0^{1,0}(\Im_T)$, anywhere.

All further considerations related to the initial boundary value problem are represented in equation (4-7)

$$\frac{\partial y(x,t)}{\partial t} - \frac{\partial}{\partial x_\kappa}\left(a_{\kappa\iota}(x)\frac{\partial y(x,t)}{\partial x_\iota}\right) + b(x)y(x,t) = f(x,t), \quad (4)$$

$$y|_{t=0} = \varphi(x), \quad (5)$$

$$y|_{x\in\partial\Im_T} = 0, \quad (6)$$

In space $\widetilde{W}_0^{1,0}(\Im_T)$;$f(x,t) \in L_{2,1}(\Im_T)$

Here,$L_{2,1}(\Im_T)$ it consists of the functions$v(x,t) \in L_1(\Im_T)$, $PvP_{L_{2,1}(\Im_T)} = \int_0^T (\int_\Im v^2(x,t)dx)^{1/2} dt$) $\frac{\partial}{\partial x_\kappa}\left(a_{\kappa\iota}(x)\frac{\partial y}{\partial x_\iota}\right) = \sum_{\kappa,\iota=1}^{n} \frac{\partial}{\partial x_\kappa}\left(a_{\kappa\iota}(x)\frac{\partial y}{\partial x_\iota}\right)$. (7)

Definition 5. The generalized solution of problems (4)–(6) is a function $y(x,t) \in \widetilde{W}_0^{1,0}(\Im_T)$ for which a relation exists and represented in equation (8-9)

$$\int_{\Im_T} y(x,t)\frac{\partial \eta(x,t)}{\partial t} dxdt + \ell_T(y,\eta) = \int_\Im \varphi(x)\eta(x,0)\,dx + \int_{\Im_T} f(x,t)\eta(x,t)\,dx\,dt \quad (8)$$

$\forall\eta(x,t) \in \widetilde{W}_0^1(\Im_T$, moreover $\eta(x,T) =$, where $\ell_T(y,\eta) = \int_{\Im_T}\left(\sum_{\kappa,\iota=1}^{n} a_{\kappa\iota}(x)\frac{\partial y(x,t)}{\partial x_\iota}\frac{\partial \eta(x,t)}{\partial x_\kappa} + b(x)y(x,t)\eta(x,t)\right) dxdt.$ (9)

3. Results and discussion

3.1 Sustainability of the differential system

This paper considers an initial boundary value problem with Dirichlet boundary conditions. All estimates, reasonings, and statements also remain valid for boundary conditions in a more general form in equation 10.

$$(a(x)\frac{\partial y}{\partial x} + \sigma y)|_{x\in\partial\Im_T} = 0, \quad (10)$$

Where $\sigma > 0$ is a constant that determines the quantitative characteristics of flows passing through the boundaries of the network carrier?

The approach presented by the results in this work can be used to obtain conditions for asymptotic or exponential sustainability of a generalized solution of differential systems. The network-like structure can be shown in Fig. 2.

The authors consider Graphthe Γ corresponding to a network-like region $\Im$ in Fig. 3 and the surface types in Fig. 4.

Let us pre-establish the solvability conditions for initial boundary value problems (2)–(4) (from now on referred to as "the differential system"), using its reduction to a differential-difference system.

Where: $\omega_1 : S_1 = S_{11} \cup S_{12}$; $\omega_2 : S_2 = S_{21} \cup S_{22} \cup S_{23}$;

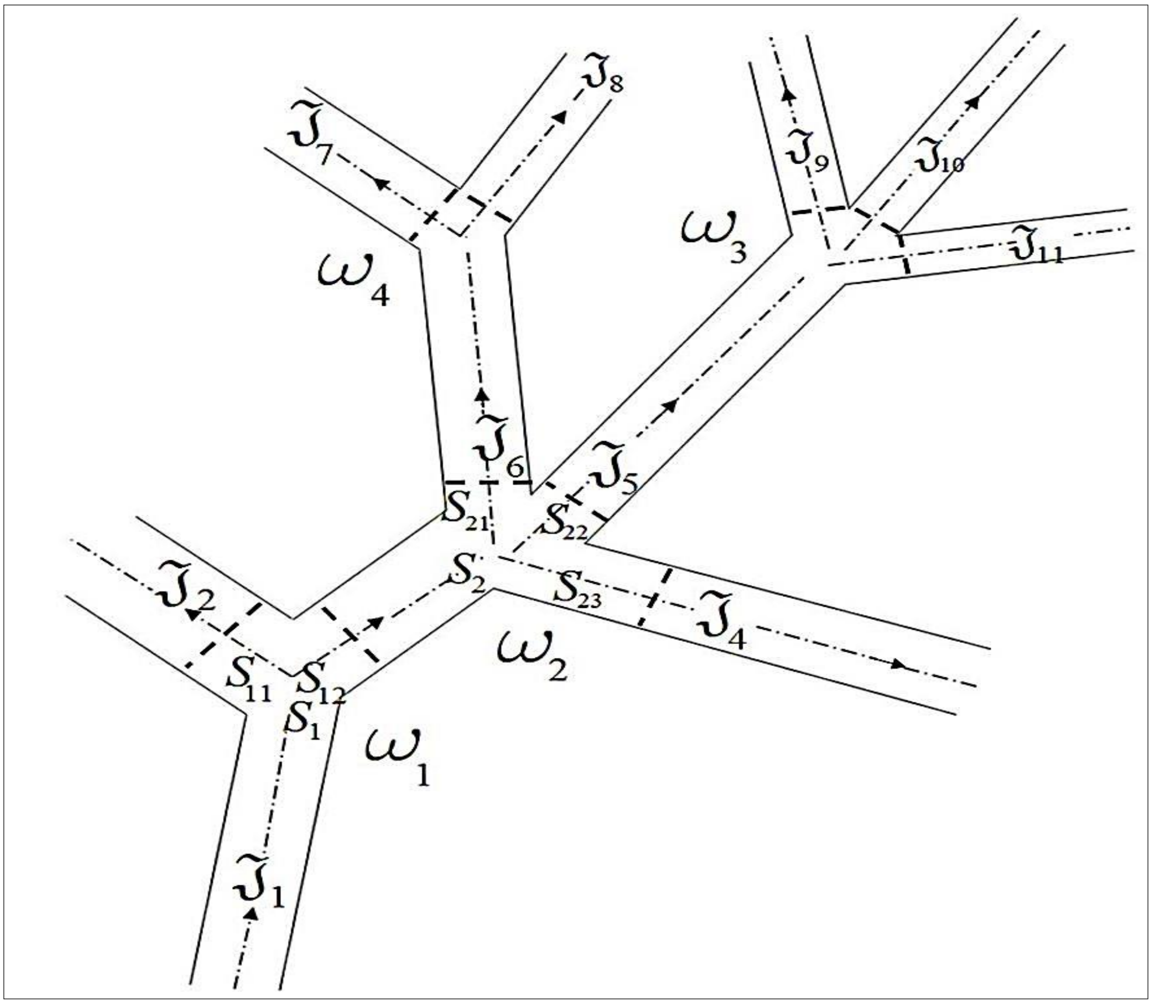

Fig. 2 Task visualization.

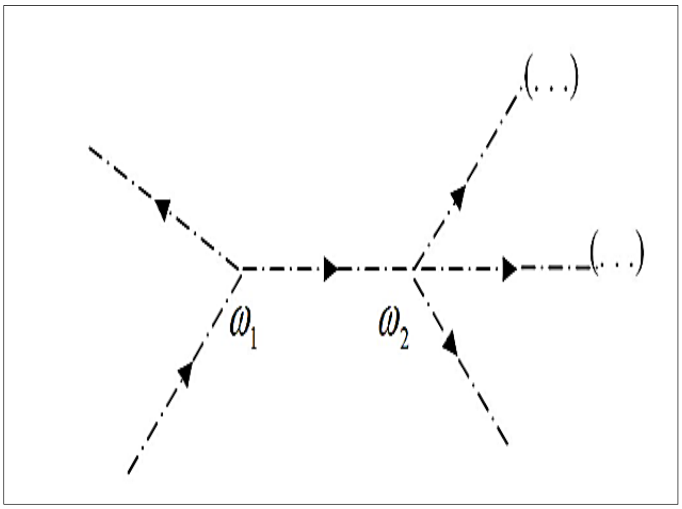

Fig. 3 Graph Γ corresponding to a network-like region $\Im$.

$\omega_3 : S_3 = S_{31} \cup S_{32} \cup S_{33};\ \omega_4 : S_4 = S_{41} \cup S_{42}$
$S_1 \subset \partial\Im_1, S_{11} \subset \partial\Im_2, S_{12} \subset \partial\Im_3,$
$S_2 \subset \partial\Im_3, S_{21} \subset \partial\Im_6, S_{22} \subset \partial\Im_5, S_{23} \subset \partial\Im_4,$ *etc.*
The authors suggested the interpretation of the n-D structure in Fig. 5.
$[0,T]$ we define a uniform grid $T^\tau = \{t_k = k\tau, k = 0,1,\dots,K\}\tau = T/K\ T^\tau$ for the segment where the grid step is.

For functions $y(k) := y(x;k) \in \widetilde{W}_0^1(\Im)\ k = 0,1,\dots,K,$ we introduce designations $y(k)_t = \frac{1}{\tau}[y(k) - y(k-1)]$, taking into account changes in the index$k\ \ k = 1,2,\dots,K$: To differential systems (2)–(4), considered in space $\widetilde{W}_0^{1,0}(\Im_T)$, we put the differential-difference system in correspondence equation (11-12)

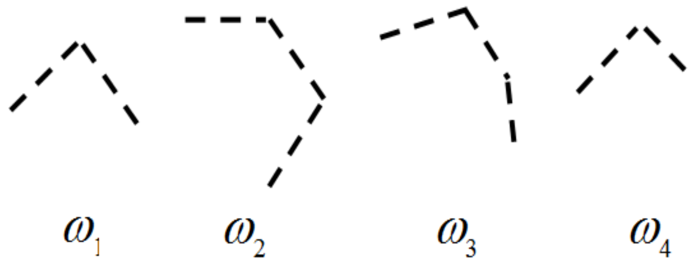

Fig. 4 Surfaces types.

$$y(k)_t - \frac{\partial}{\partial x_\kappa}\left(a_{\kappa\iota}(x)\frac{\partial y(k)}{\partial x_\iota}\right) + b(x)y(k) = f_\tau(k),$$
$$k = 1,2,\dots,K, \tag{11}$$
$$y(0) = \varphi(x) \in L_2(\Im), y(k)|_{x\in\partial\Im} = 0, \tag{12}$$

In space $\widetilde{W}_0^1(\Im)$, functions $f_\tau(k) := f_\tau(x;k)\ (k = 1,2,\dots,K)$ from (5) and $f(x,t)$ from (2) are connected by relations:

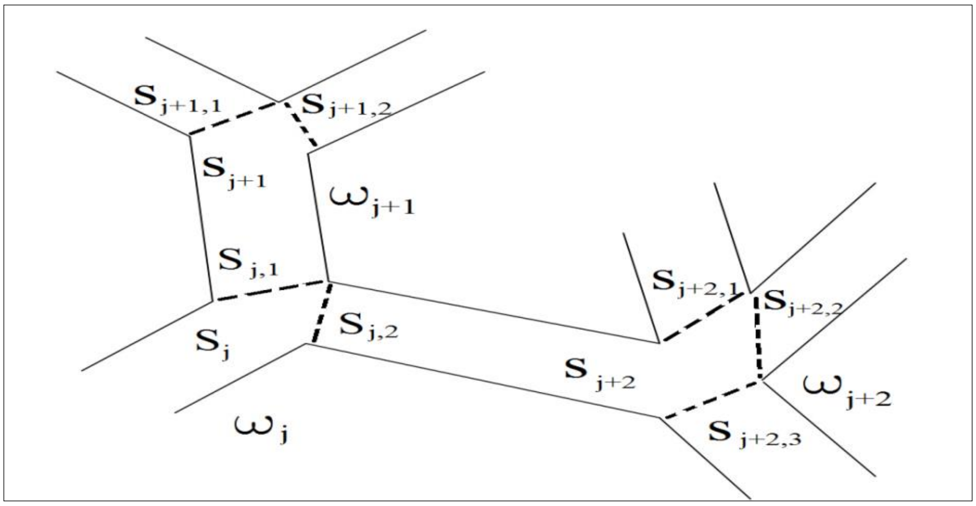

Fig. 5 Interpretation in the most general form.

$f_\tau(x;k) = \frac{1}{\tau}\int_{(k-1)\tau}^{k\tau} f(x,t)dt \in L_2(\Im) k = 1,2,\dots,K,$

Below (see the proof of Theorem 1), we show the role of the differential-difference system (5) and (6) in obtaining the solvability conditions for differential systems (2)–(4).

Note 3. The difference relation $y(k)_t = \frac{1}{\tau}[y(k) - y(k-1)]$ used in relation (5) determines the approximation overtime variable t of partial derivative $\frac{\partial y(x,t)}{\partial t}$, the error of which is equal to $O(\tau)$ relation (5) and determines the two-layer semi-discretization of the differential system (2). It can use another, more accurate approximation for expression $\frac{\partial y(x,t)}{\partial t}$; for example, a three-layer $\frac{1}{2\tau}[3y(k) - 2y(k-1) + y(k-2)]$, the error of which is equal $O(\tau^2)$. In this case, it is necessary to supplement the first relation in (6) with the relation $y(1) = \varphi_1(x) \in L_2(\Im)$.

Definition 6. Collection $y(k) \in \widetilde{W}_0^1(\Im)$ $(k = 1,2,\dots,K)$ is a generalized solution of systems (5) and (6) if, for any fixed k $(k = 1,2,\dots,K)$, the function $y(k)$ satisfies the relation.

$$\int_{\Im} y(k)_t v(x)dx + \ell(y(k),v) = \int_{\Im} f_\tau(k)v(x)dx \forall v(x) \in \widetilde{W}_0^1(\Im).$$

In this case, equality $y(0) = \varphi(x)$ in (4) is defined almost anywhere, and the form of $l(y(k),v)$ is:

$$\ell(y(k),v) = \int_{\Im}\left(\sum_{\kappa,\iota=1}^{n} a_{\kappa\iota}(x)\frac{\partial y(x;k)}{\partial x_\iota}\frac{\partial v(x)}{\partial x_\kappa} + b(x)y(x;k)v(x)\right)dx.$$

It is easy to see those relations (5) and (6), for any fixed k $(k = 1,2,\dots,K)$, determine the boundary value problem $\widetilde{W}_0^1(\Im)$, and, for this problem, the following statement is true.[22,23]

Lemma. Let factors $a_{\kappa\iota}(x)$ $b(x)$ of the system (2) be subject to the conditions as shown in equations (13-15).

$$a_{\kappa\iota}(x) \in L_2(\Im)(\kappa,\iota = 1,2,\dots,n), b(x) \in L_2(\Im), \quad (13)$$

$$a_{\kappa\iota}(x) = a_{\iota\kappa}(x), |b(x)| \leq \beta, \quad (14)$$

$$a_*\bar{\xi}^2 \leq a_{\kappa\iota}(x)\xi_\kappa\xi_\iota \leq a^*\bar{\xi}^2, \bar{\xi} = (\xi_1,\xi_2,\dots,\xi_n,), \quad \bar{\xi}^2 = \sqrt{\sum_{\kappa=1}^{n}\xi_\kappa^2}, \quad (15)$$

a_*, a^*, β are specified positive constants and $\xi_1,\xi_2,\dots,\xi_n$ are arbitrary parameters. Then, any fixed k $(k = 1,2,\dots,K)$ boundary value problems (5) and (6) in space $\widetilde{W}_0^1(\Im)$ are uniquely solvable in a generalized sense for a sufficiently small value τ. When $\tau \leq \tau_0 < \frac{1}{4\beta}$, for the functions of the collection $y(k) \in \widetilde{W}_0^1(\Im)$ $(k = 1,2,\dots,K)$, the following inequalities are actual equation (16-17)

$$\| y(k) \|_{2,\Im} \leq e^{4\beta T}\left(\| \varphi \|_{2,\Im} + 2 \| f_\tau(k) \|_{2,1,\Im}\right), \quad (16)$$

And for $m = 1,2,\dots,K$:

$$P\, y(m)\, P_{2,\Im}^2 + 2a_*\tau\sum_{k=1}^{m} P\frac{dy(k)}{dx}P_{2,\Im}^2 + \tau^2\sum_{k=1}^{m} P\, y(k)_t\, P_{2,\Im}^2 \leq C(P\varphi P_{2,\Im}^2 + P f_\tau(m) P_{2,1,\Im}^2), \quad (17)$$

Here $P \cdot P_{2,\Im}$ is the norm $L_2(\Im)$ $P\, f_\tau(m)\, P_{2,1,\Im} = \tau \sum_{k=1}^{m} P\, f(k) P_{2,\Im}$ $C > 0$, not dependent on τ.

The following statement defines the solvability conditions for differential systems (2)–(4).

Theorem 1. If the conditions of the lemma are satisfied, then differential systems (2)–(4) have a generalized solution belonging to space $\widetilde{W}_0^{1,0}(\Im_T)$.

Proof. Let us consider differential-difference systems (5) and (6). Given the fulfilment of conditions (7)–(9) and the statement of the lemma, boundary value problems (5) and (6) for each fixed k $(k = 1,2,\dots,K)$ are solvable, and the generalized solution is $y(k) \in \widetilde{W}_0^1(\Im)$. Based on the resulting collection $y(k) \in \widetilde{W}_0^1(\Im)$ $(k = 1,2,\dots,K)$, we construct a function $y_K(x,t)$ of variables $x,t \in \Im_T$ according to the following formula in equation (18)

$$y_K(x,t) = y(k), t \in ((k-1)\tau, k\tau], k = 1,2,\dots,K. \quad (18)$$

It should be noted that function $y_K(x,t)$ in relation (12) is a piecewise constant interpolation over a variable $t \in (0,T]$ using the functions $y(k)$ $(k = 1,2,\dots,K)$. Similar interpolations will be introduced for all functions of variables $x, t \in \Im_T$ used in the proof of the theorem. Verifying that the function $y_K(x,t)$ belongs to space $\widetilde{W}_0^{1,0}(\Im_T)$ and those inequalities (10) and (11) are valid is easy. The latter means the validity of the inequality in equation (19)

$$\|y_K\|_{2,\Im_T} + \|\frac{\partial y_K}{\partial x}\|_{2,\Im_T} \le C^* \quad (19)$$

(using $\|\cdot\|_{2,\Im_T}$ norm $L_2(\Im_T)$ is denoted); $C^* > 0$ in (13) depends only on a_*, β and T. Next, we construct the function $f_K(x,t)$ as a piecewise constant interpolation over a variable $t \in (0,T]$ using functions $y(k)$ $(k = 1,2,\ldots,K)$:

$$f_K(x,t) = f_k(k), t \in ((k-1)\tau, k\tau], k = 1,2,\ldots,K. \quad (20)$$

By changing (increasing) the number K in relations (5) and (6) as shown in Equation (20) we obtain an extension of collection $y(k) \in \widetilde{W}_0^1(\Im)$ $(k = 1,2,\ldots,K)$, which generates a functional sequence $\{y_K(x,t)\}$. Note, in this case, that norms $\| y_K\|_{2,\Im_T}\|\frac{\partial y_K}{\partial x}\|_{2,\Im_T}$ satisfy (13) for any $K = 1,2,\ldots$. This means that $\{y_K(x,t)\}$ it includes a subsequence $\{Y_{K_s}(x,t)\}$ that weakly converges on the norm of space $\widetilde{W}_0^{1,0}(\Im_T)$ with an element $Y(x,t)$ of the same space. For a complete proof of the theorem, it is enough to demonstrate that the specified element $Y(x,t)$ satisfies the integral identity of Definition 5 for any pre-selected function $\eta(x,t)$ from space $\widetilde{W}_0^1(\Im_T)$ $\eta(x,T) = 0$. Let us denote, using $\tilde{C}^1(\bar{\Im}_{T+\tau})$, the set of functions $\eta(x,t)$ differentiable in the region $\bar{\Im}_{T+\tau}$ $\Im_{T+\tau} = \Im \times (0, T+\tau)$, such that they satisfy relations $\eta|_{\partial\Im_T} = 0 \eta|_{t\in[T,T+\tau]} = 0$; it is clear that $\tilde{C}^1(\bar{\Im}_{T+\tau})$ the set is tightly included $\widetilde{W}_0^1(\Im)$. Given the continuity $\eta(x,t)$, we define functions $\eta(k)$ using the following relations:

$\eta(k) = \eta(x, k\tau), k = 1,2,\ldots,K,$

We define the right $\eta(k)_{t'} = \frac{1}{\tau}[\eta(k+1) - \eta(k)]$ and the left $\eta(k)_t = \frac{1}{\tau}[\eta(k) - \eta(k-1)]$ difference relations, respectively, which are approximations over time variable t s of the derivative $\frac{\partial\eta}{\partial t}$ at a point $t = k\tau$. Similar to the construction of functions $y_K(x,t)$ $f_K(x,t)$ and, according to the obtained $\eta(k)$, functions $\eta_K(x,t) \frac{\partial\eta_K(x,t)}{\partial x} \frac{\partial\eta_K(x,t)}{\partial t}$ are constructed. The uniform convergence for $K \to \infty \bar{\Im}_T$ sequences $\{\eta_K(x,t)\}\left\{\frac{\partial\eta_K(x,t)}{\partial x}\right\}and\left\{\frac{\partial\eta_K(x,t)}{\partial t}\right\}$ elements $\eta(x,t)$ $\frac{\partial\eta(x,t)}{\partial x}\frac{\partial\eta(x,t)}{\partial t}$ is evident; the relation $\eta_K(x,t) = 0 t \in [T, T+\tau]$ is also apparent. Replacing functions $y(x,t)$ $f(x,t)$, and $\eta(x,t)$ in the identity of Definition 5 with $y_K(x,t)$ $f_K(x,t)$ $\eta_K(x,t)$, and then passing along subsequence $\{Y_{K_s}(x,t)\}$ in the obtained relation to the limit leads to the following: the limit function $Y(x,t) \in \widetilde{W}_0^{1,0}(\Im_T)$ satisfies the relation presented in Definition 5 for any $\eta(x,t) \in \tilde{C}^1(\bar{\Im}_{T+\tau})$. Hence, the function $Y(x,t) \in \widetilde{W}_0^{1,0}(\Im_T)$ is a generalized solution of differential systems (2)–(4), i.e., the theorem statement is ultimately proved.

To simplify the presentation, we will reduce the space $L_{2,1}(\Im_T)$ below by choosing the right parts $f(x,t)$ of the relation (2). Namely, we replace $L_{2,1}(\Im_T)$ with $CL_{2,1}(\Im_T) \subset L_{2,1}(\Im_T)$ and assume that $f(x,t) \in CL_{2,1}(\Im_T)$ (elements $CL_{2,1}(\Im_T)$ are functions $L_{2,1}(\Im_T)$ that are continuous over a variable t in norm $L_2(\Im)$) (such a replacement is natural in problems of applied analyses). When studying the sustainability property of a generalized solution $y(x,t)$ of systems (2)–(4), the behavior $y(x,t)$ with an unlimited increase in t, i.e., with $t \in [0,\infty)$, is essential. The following statement is true.[24]

Theorem 2. Assume that $f(x,t) \in CL_{2,1}(\Im_T)$ satisfies the condition $\int_t^{t+1} \|f(\cdot,\varsigma)\|_{2,\Im}^2 d\varsigma \le A A > 0$ and is a constant. Then, for the generalized solution $y(x,t)$ of differential systems (2)–(4), there is a constant $C > 0$ such that:

a) $\int_t^{t+1} \|y(\cdot,\varsigma)\|_{W_2^1(\Im)}^2 d\varsigma \le C \forall t \ge 0$ b) $\|y(\cdot,t))\|_{2,\Im} \le C, t \to +\infty$.

Let us assume that the condition $0 \le b(x) \le \beta$ $x \in \Im$for the function $b(x)$ is satisfied instead of (8), which is also natural in applications. This ensures positive eigenvalues of $\lambda_i, i \ge 1$, fer the elliptic operator in equation (20)

$$\Re u = -\frac{\partial}{\partial x_\kappa}\left(a_{\kappa\iota}(x)\frac{\partial u(x)}{\partial x_\iota}\right) + b(x)u(x)$$

Which is defined in space $\widetilde{W}_0^1(\Im)$, and also, given the statements of Theorem 2, the representation of the generalized solution $y(x,t)$ of differential systems (2)–(4) in the form of a series:

$$y(x,t) = \sum_{i=1}^{\infty}\left(\varphi_i e^{-\lambda_i t} + \int_0^t f_i(\zeta)e^{-\lambda_i(t-\zeta)}d\zeta\right)u_i(x) \quad (20)$$

According to the system of generalized eigenfunctions, $\{u_i(x)\}$ the operator $\Re$:

$$\varphi(x) = \sum_{i=1}^{\infty}\varphi_i u_i(x), \varphi_i = \int_{\Im}\varphi(x)u_i(x)dx,$$

$$f(x,t) = \sum_{i=1}^{\infty} f_i(t)u_i(x), f_i(t) = \int_{\Im} f(x,t)u_i(x)dx, t \in [0,T].$$

Next, $\Im_\infty = \Im \times (0,\infty)$ by changing variables x t in the region, we consider differential systems (2)–(4). We introduce the following designations: $\Im_{t_0,t} = \Im \times (t_0,t)$ $\partial\Im_{t_0,t} = \partial\Im \times (t_0,t)$ $0 < t_0 < t < \infty \Im_{t_0,\infty} = \Im \times (t_0,\infty)$ $\partial\Im_{t_0,\infty} = \partial\Im \times (t_0,\infty)$ (), obviously, $\Im_{t_0,t} \subset \Im_t$

Let us denote $\bar{y}(x,t) \in \widetilde{W}_0^{1,0}(\Im_{t_0,\infty})$ as the state of the system (2), which is a generalized solution (17) that satisfies the conditions:

$$y|_{t=t_0} = \bar{\varphi}(x), \quad x \in \Im, \quad y|_{x\in\partial\Im_{t_0,\infty}} = 0, \quad (21)$$

Let us denote $y(x,t) \in \widetilde{W}_0^{1,0}(\Im_{t_0,\infty})$ as the state of system (2), which is a generalized solution (2) that satisfies similar conditions in equation (21-22).

$$y|_{t=t_0} = \varphi(x), \quad x \in \Im, \quad y|_{x\in\partial\Im_{t_0,\infty}} = 0, \quad (22)$$

The differences between initial boundary value problems (2) and (15), as well as (2) and (16), are only functions $\bar{\varphi}(x)$ $\varphi(x)$ in the boundary conditions of relations (15) and (16), respectively. By the accepted terminology of the sustainability theory, we will call the solution $\bar{y}(x,t)$ the undisturbed state of the system (2) and $y(x,t)$ the disturbed state of the system (2). From the statements of Theorem 2 and representation (14) of a generalized solution of system (2),

it follows that functions $\bar{y}(x,t)$ $y(x,t)$ are defined for $x, t \in$

$\Im_{t_0,\infty}$, and the initial and boundary conditions (15) and (16) are defined for them; moreover, $\bar{y}(x,t)$ $y(x,t)$ they are elements of space $\widetilde{W}_0^{1,0}(\Im_{t_0,\infty})$ $f(x,t) \in CL_{2,1}(\Im_\infty)$ for.

Definition 7. If $\delta(t_0,\varepsilon) > 0$it exists for any arbitrary $t_0 > 0\varepsilon > 0$and, such that inequality $Py(\cdot,t) - \bar{y}(\cdot,t)\, P_{\widetilde{W}_0^1(\Im)} < \varepsilon$ is satisfied for $P\varphi - \bar{\varphi}\, P_{L_2(\Im)} < \delta(t_0,\varepsilon)$ and for $t \geq t_0$, then the undisturbed state $\bar{y}(x,t)$ of the system (2) is called sustainable; here $y(x,t)$ is the disturbed state of the system (2).

Theorem 3. If the conditions of Theorem 2 are satisfied, then the $\bar{y}(x,t)$ system's state (2) is sustainable $\Im_{t_0,\infty}$.

Proof. Let us consider function $\theta(x,t) = y(x,t) - \bar{y}(x,t)$. Since system (2) is linear and both functions $\bar{y}(x,t)$ $\bar{y}(x,t)$ belong to $\widetilde{W}_0^{1,0}(\Im_{t_0,\infty})$, we obtain $\theta(x,t) \in \widetilde{W}_0^{1,0}(\Im_{t_0,\infty})$ a generalized solution of homogeneous differential system (2) for $f = 0$, in addition, the following conditions are satisfied in equation (23).

$$\theta|_{t=t_0} = \phi(x), \quad x \in \Im, \quad \theta|_{x\in\partial\Im_{t_0,\infty}} = 0, \tag{23}$$

Where $\phi(x) = \varphi(x) - \bar{\varphi}(x)$. Reasonings similar to those in the proof of Theorem 2 lead to the establishment of the solvability of differential systems (2) and (17), and its solution has the form:

$$\theta(x,t) = \sum_{i=1}^{\infty} \phi_i e^{-\lambda_i t} u_i(x), \phi_i = \int_{\Im} \phi(x) u_i\,(x) dx,$$

Being a weak limit of the sequence $\{\theta^N\}_{N\geq 1}$ of functions $\theta^N(x,t) = \sum_{i=1}^{N} \phi_i e^{-\lambda_i t} u_i(x)$ $N \to \infty$. Note that for norms $P\theta^N P_{2,\Im_t}$, the following inequality is true:

$$P\,\theta^N P_{2,\Im_t} \leq \sum_{i=1}^{N} \phi_i^2 e^{-2\lambda_i t} \quad \forall N = 1,2,\ldots$$

Passing the limit for $N \to \infty$ this inequality, we obtain:

$P\theta P_{2,\Im_t} \leq C^* P\phi P_{2,\Im} \forall t \in [t_0,\infty)$

(here, the property $e^{-2\lambda_i t} < 1$ $i = 1,2,\ldots$ is taken into account), where C^* is a constant that is independent t From the obtained inequality, it follows in equation (24)

$$\|\theta(\cdot,t)\|_{\widetilde{W}_0^1(\Im)} \leq C^*\, P\phi\, P_{2,\Im}. \tag{24}$$

Let us fix $\varepsilon > 0$ and assume that $\delta = \frac{\varepsilon}{C^*}$, then, from (18), we obtain:

$P\phi P_{2,\Im} = P\varphi - \bar{\varphi}\, P_{2,\Im} < \delta,$

And hence inequality $P\,\theta(\cdot,t)\, P_{\widetilde{W}_0^1(\Im)} = Py(\cdot,t) - \bar{y}(\cdot,t)\, P_{\widetilde{W}_0^1(\Im)} < \varepsilon$ for an arbitrary $t > t_0$.

The theorem is proved entirely. When analyzing the stability of a generalized solution of a differential system, the classical approach used to analyze the stability is used to analyze the stability of solutions to ordinary differential equations. Taking into account the linearity of the elliptic part of the parabolic equation, a weak solution of this equation under certain conditions on the influence function on the right side of the equation (these conditions are not burdensome in applications) can be represented as expansions in an exceptional basis, the set of generalized eigenfunctions of the elliptic operator.[25-27] The expansion coefficients are exponential functions with exponents determined by the product of the eigenvalues of the elliptic operator (the eigenvalues are harmful due to the complete continuity of this operator). The latter guarantees the technical stability of a weak solution for a sufficiently long time interval in applied situations and classical stability (Lyapunov stability) on an infinite time interval (the latter in the case of preserving its properties at infinity of a function characterizing external influences). The authors note that ways to analyze the asymptotic stability are open here. Examples to illustrate the classical method are self-evident. The proposed model has been compared with the existing A.H.P. (Analytic Hierarchy Process), D.E.A. (Data Envelopment Analysis), M.C.D.M. (Multi-Criteria Decision Making) and L.S.S.V.M. (Least Squares Support Vector Machines).

3.2 Resource availability

The sustainability analysis of energy resources transport based on a digital N-D logistics network provides an innovative approach to understanding resource availability better. The analysis helps to identify where and when resources can be most efficiently used. Shippers and receivers can view and analyze data relevant to their needs and decisions using the digital network. This also helps to quickly determine if the transportation of resources to and from specific locations is more efficient than other methods. Analyzing resource availability for the N-D logistics network can help parties in the energy production and transportation industry make more informed decisions about utilizing their resources as shown in Fig. 6.

This analysis also helps identify possible opportunities for cost savings and increased efficiency while ensuring that the resources' sustainability lasts longer. Additionally, the data in the analysis can give insight into how resources are used in the current infrastructure and if any adjustments need to be made to ensure efficient and sustainable energy transportation. Overall, resource availability analysis for the digital N-D logistics network provides an excellent tool for stakeholders in the energy industry to identify and utilize resources more efficiently and sustainably. The analysis can also help them make more informed decisions about the transportation and use of their resources.

3.3 Battery life

Analyzing battery life for the sustainability of energy resources transport based on digital N-D logistics networks is particularly important for ensuring that the operations are low-cost and reliable. Battery performance is a critical factor in this regard, as it can impact the energy efficiency of the transport process. Battery life is reduced over time due to various factors such as age, temperature and charge cycles. An analysis of battery life shown in Fig. 7 in this context must start with a detailed assessment of the environmental conditions, including temperature, humidity, and usage duration.

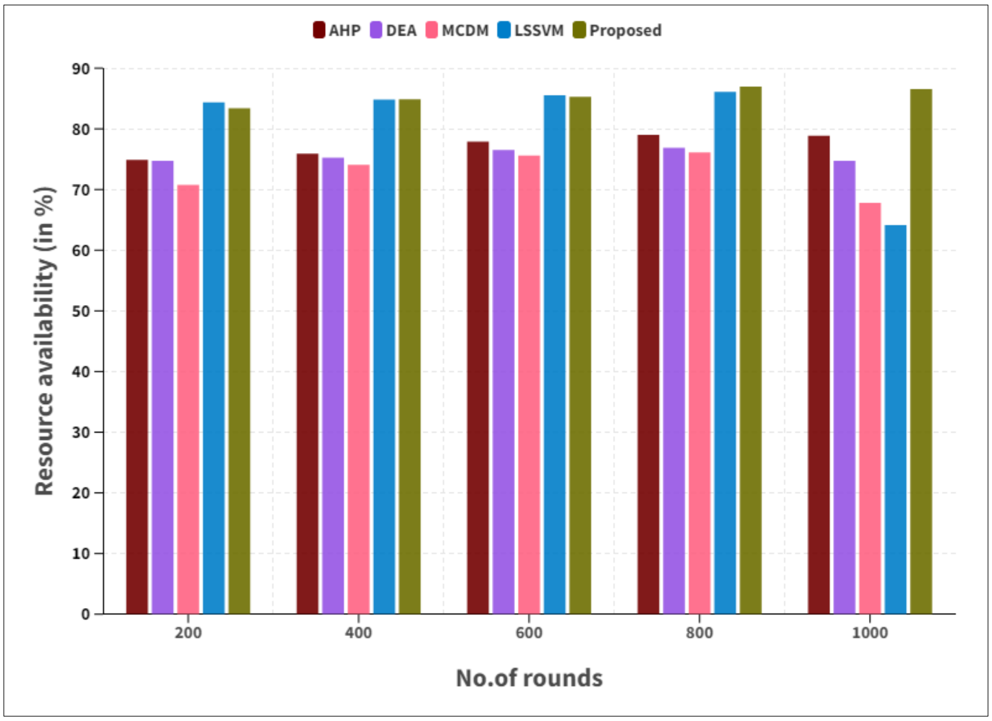

Fig. 6 Comparison of resource availability.

It should also include an evaluation of the available charging and discharging options (e.g., direct or alternating current). This analysis should understand the current battery life and suggest ways of improving battery performance, mainly through advanced technology. Finally, the application of preventive, predictive, and corrective maintenance strategies should be considered to improve battery life further and reduce the cost of energy resource transportation.

3.4 Energy storage capacity

To analyze the performance of energy storage capacity, an analysis of the types of energy resources stored, the cost of storage, the safety of stored resources, and the efficiency of the storage system is shown in Fig. 8. Different types of energy resources require different storage methods, as some require battery storage while others require higher-capacity tanks or underground storage. The cost of energy storage depends on the type of storage technology chosen, which must balance performance and cost factors. Concerning safety, proper safety measures should consistently be implemented to ensure that the stored energy resources are safe for use.

Finally, the efficiency of the storage system should be evaluated concerning reliability, scalability, and overall energy efficiency. Overall, analyzing the performance of energy

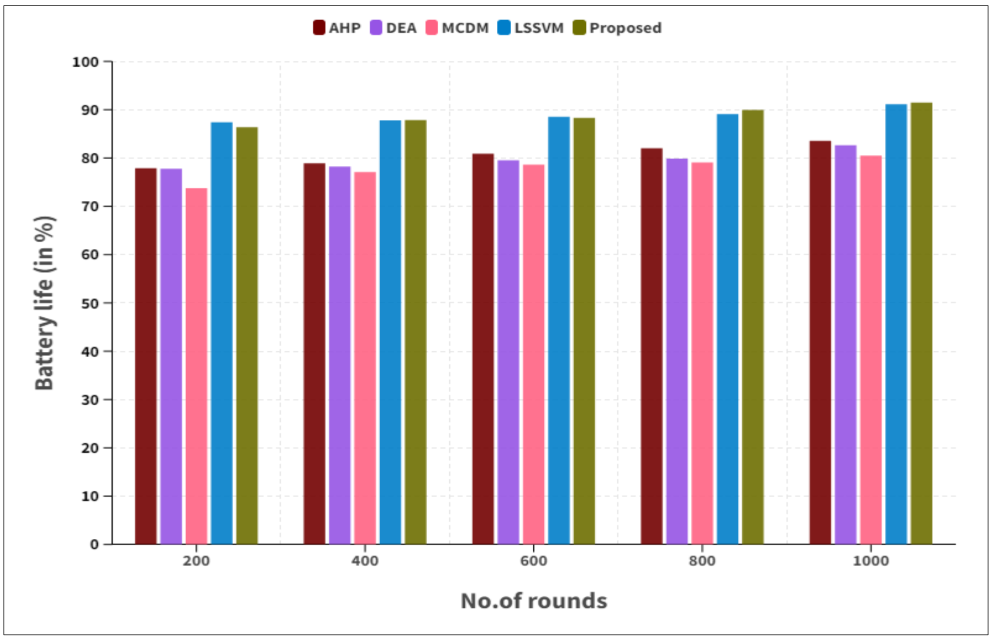

Fig. 7 Comparison of battery life.

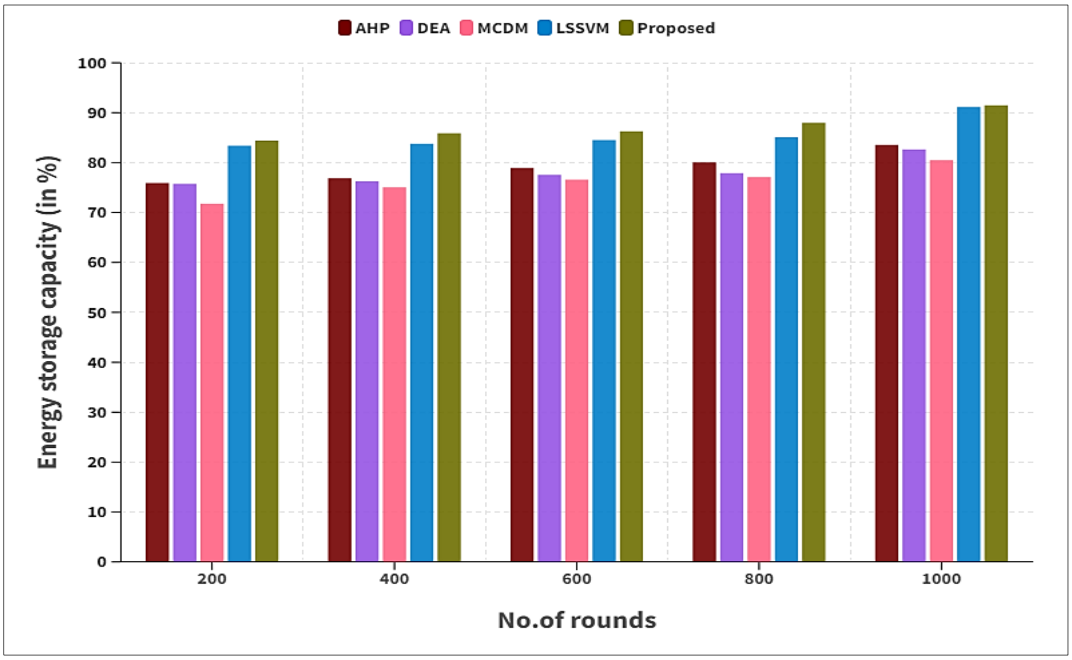

Fig. 8 Comparison of energy storage capacity.

storage capacity is an essential step in assessing the sustainability of energy resource transport using a digital N-D logistics network. By considering all the factors associated with energy storage capacity, such as the type of energy stored, cost of storage, safety of stored resources, and the efficiency of the storage system, organizations can ensure that energy resource transport is reliable, stable, and resourceful.

3.5 Energy efficiency

The Energy efficiency for sustainable analysis of energy resources transport is shown in Fig. 9 and is based on a Digital N-D Logistics Network that has been positively evaluated. The digital N-D Logistics Network (DLN) can create optimized pathways and routes that enable the collection, distribution and transport of energy resources in an energy-efficient manner. The use of DLN has helped to reduce energy consumption by minimizing the number of vehicles used, number of trips and waiting times. Furthermore, the DLN has helped to reduce pollution levels by minimizing fuel consumption.

The DLN has also helped increase efficiency across the supply chain by providing organizations with more options, optimizing multiple supply chains and promoting sustainable procurement practices. Incorporating ICT-based technologies enables the establishment of dynamic multi-objective resources and energy modeling, which can help identify the most energy-efficient routes. In addition, real-time energy monitoring and measurement of the DLN assists with operating energy resources in an optimized manner. The performance comparison is shown in the following Table 1.

Figure 10 shows, the proposed model reached 85.29% resource availability, 88.29% battery life, 86.29% energy storage capacity, and 91.29% energy efficiency. In the same range, the existing A.H.P. reached 77.89% Resource availability, 80.89% Battery life, 78.89% Energy storage capacity and 82.89% Energy efficiency, D.E.A. reached 76.53% Resource availability, 79.53% Battery life, 77.53% Energy storage capacity and 81.53% Energy efficiency, M.C.D.M. reached 75.60% Resource availability, 78.60% Battery life, 76.60% Energy storage capacity and 80.60% Energy efficiency, L.S.S.V.M. reached 85.02% Resource availability, 88.12% Battery life, 86.29% Energy storage capacity and 91.29% Energy efficiency.

Table 1. Performance comparison.

Parameters	AHP	DEA	MCDM	LSSVM	Proposed
Resource availability [R.A.]	77.89	76.53	75.60	85.02	85.29
Battery life [B.L.]	80.89	79.53	78.60	88.12	88.29
Energy storage capacity [SC]	78.89	77.53	76.60	84.52	86.29
Energy efficiency [E.E.]	82.89	81.53	80.60	89.52	91.29

4. Discussion

The authors attempt to develop a methodological approach to analyze the sustainability of the transport flows of energy resources, considering the physical essence of transported flows (this is particularly important in studying the transfer of multiphase media). The researchers propose extending their approach to improving well-known analytical views on modeling transportation flows in various fields[28–30] by considering dynamic processes in network-like multidimensional structures. From the author's point of view,

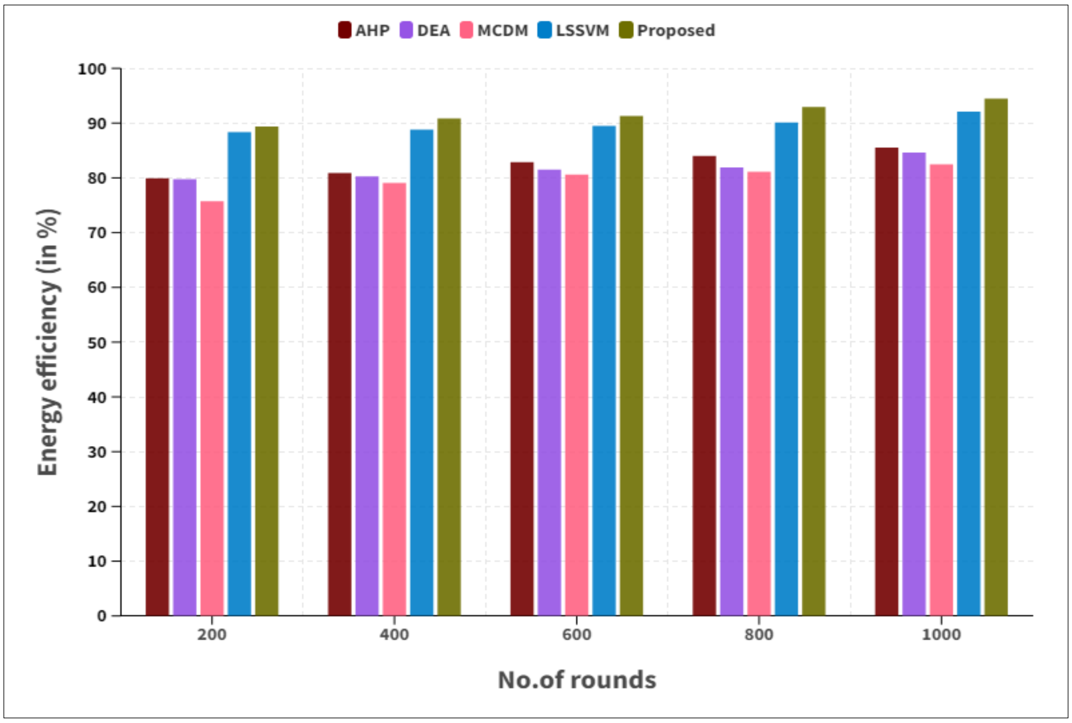

Fig. 9 Comparison of energy efficiency.

the digital transformations have led to a subsequent transition from traditional networks to designing complex networks[31–34] based on multidimensional interactions within digital ecosystems. The main idea is that the n-D methodology can be a fundamental base for developing synchronized interactions in customer-centered businesses, especially in international logistics and marketing channels. The authors suggested various methodological approaches to designing digital marketing omnichannel networking with clients. The developed approach could allow for organizing seamless interactions with customers.[35] Calancea and Alboaie[36] explored customer service processes based on the methodology of multidimensional Markov chains. Limiting the flow of requests that exceed a certain threshold ensures servicing clients in the network. The client receives the requested service if the request's arrival does not exhaust the network bandwidth. The work in question proposes considering customer impatience and provides recommendations for improving network performance by upgrading the bottleneck node. RFID model that considers the interaction between the seller and the buyer after the sale.[37-40] According to the developers of the model, the novelty of RFID lies in the analysis of the value of the client from the point of view of the contact centre. The model is based on a double fuzzy linguistic model, the R.F.M. model, and an analytical hierarchical process (A.H.P.). The model can help develop user profiles and personalize messages, including artificial intelligence algorithms.

The authors propose to pay attention to the issue of supplier selection in the framework of supply chain management in the work of Kwilinski *et al*.[41] they developed a hybrid multi-criteria decision model (M.C.D.M.) using the fuzzy analytic hierarchy process (F.A.H.P.) and the green data envelopment analysis (G.D.E.A.) model. The hybrid model allows one to determine the weight of all the criteria in the supplier-election process based on the judgments of the company's procurement experts. From the position of business-to-business data exchange, the authors propose considering the role of open data in the innovation process.[42] Companies participating in the B2B process with suppliers and consumers have advantages in improving the quality of service. Data collected from multiple businesses make determining customer experience regarding purchased products easy. This allows us to adjust products and better match customers' expectations to satisfy them. According to the authors, developing a digital omnichannel marketing methodology is based on synchronizing interactions between the client and the service provider. The considered synchronization requires a relevant representation of the multidimensionality of the interaction process with the client. The methodology of n-dimensional structures allows for integrating digital marketing and logistics approaches. The authors propose considering sustainability issues[42,44] and the features of digital interactions within customer value chains.[39] The authors' system has been initially implemented to transport energy resources.[40] It should rely on the environmental[41] and social goals of transforming infrastructure and opening the door to new entrants.[42] Cano *et al*.[45] are looking at improving green competitiveness by implementing an omnichannel strategy based on synchronizing marketing communication channels and identifying cause-and-effect relationships between relevant quality criteria. Interestingly, the Russian industry's interest in this ecological topic is increasing steadily.[43-45] The authors develop theoretical provisions that allow for the

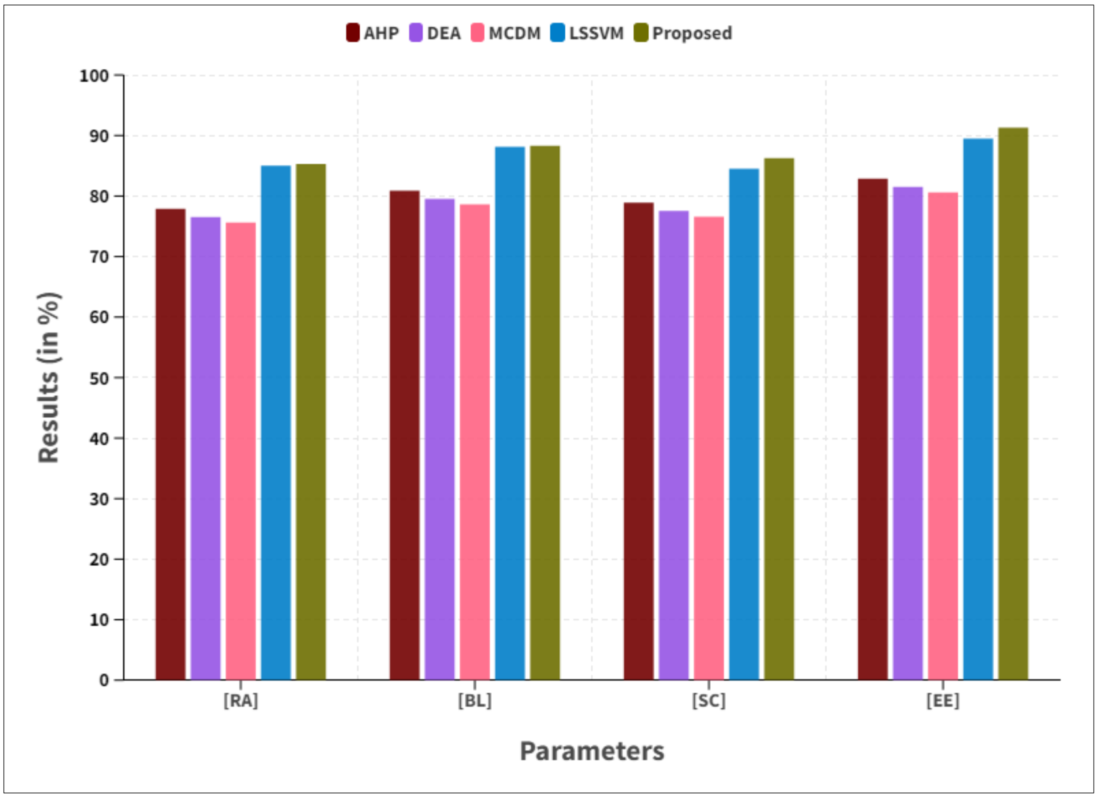

Fig. 10 Comparison of overall performance analysis.

analysis of the multidimensional structures of various media.[46-49] The study proposes an approach to forming n-dimensional structures to improve the quality of customer service based on the integration of marketing and logistics theory.

5. Conclusions

In recent years, algorithms for digital logistics platforms have progressed to a new level of quality. Science-based methods are applied using network models of n-D flows with distributed parameters. The analysis of the sustainability of transport flows of energy resources in network n-D carriers can be easily transferred to establish conditions for the sustainability of transport flows in the leading carriers of energy resources, where there are no flow branching points. The same approach applies to multidimensional functions that describe the sustainability of the state of a system under study. The results obtained are fundamental for the problems of optimal control and stabilization of differential systems, including systems with delays. Developing a 4PL provider or fourth-party logistics is influenced by algorithms and models that 3PL players widely use in the segment. Here, the transition to the next level is determined by the acceptance of obligations by a third-party logistics outsourcer to provide additional consulting services to optimize the share of the customer's expenses on logistics. Logistics includes many tasks, and transportation costs are their most expensive. In the segment of finished products, the share of transport costs is within 5–10% of the market value. A completely different picture is observed in the cost composition of raw commodities. Here, transport costs add up to 50% of the final price. Countries with critically dependent economies on trading raw materials, methods for finding optimal solutions and assessing their sustainability mean competitive advantage. The research objective of this study was to explore the efficiency and sustainability of transporting energy resources through a digital N-D Logistics network. Experiments revealed that the digital N-D Logistics network effectively optimized the shipment of energy resources and increased their sustainability. This is essential in reducing waste and improving energy efficiency during transit and shipping operations. In addition, the research indicated that the digital N-D Logistics network could be applied for longer distances without causing significant overutilization of resources and energy consumption. Further research is needed to understand the logistics network's more detailed aspects, including scalability, reliability, and feasibility of implementation. Limitations to the current research are related to the scalability and cost of the network, which may limit the usability of it for a larger-scale operation.

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Conflict of Interest

There is no conflict of interest.

Supporting Information

Not applicable.

Nomenclature

Appexdix 1: Mathematical symbols expression

Expression	Symbol
Space	$\tilde{W}_0^1(\Im)$
closure of reduction	$\tilde{C}_0^1(\overline{\Im})$
closure of collection	$\tilde{C}^1(\overline{\Im})$
A network-like region	$\Im$
Euclidean space	$\mathbb{R}^n$
Surface	S_j
union of surfaces	S_{ji}
continuous on closure	$\overline{\Omega}$
vectors	$\bar{n}_j$
Subdomains	$\Im_l\ (l = \overline{1, N})$
A network like region boundaries	$\partial\Im_l$
Network nodes	$\omega_j j = \overline{1, M}$
Number of nodes	M
Elliptic operator	$\mathcal{A}u$
Eigenfunctions	$\{\phi_i(x)\}_{i=1}^{\infty}$
Spatial variable	$x = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n$
Transfer process function	$u(x)$
Set of differentiable functions	$C^1(\overline{\Omega})$
Lebesgue space	$L_2(\Omega)$
Sobolev space	$W_2^1(\Omega)$
Derivative	$a(x)$
Network pitch	τ
Functions in the process of calculation	$f_\tau(k), O(\tau), y(k), \ell(y(k), v)$
Permanent	a_*, a^*, β
Indices	i, j, k
Parameters	$\xi_1, \xi_2, \dots, \xi_n, \nu, \rho$

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